CS660: Grad Intro to Database Systems

Class 9: External Sorting

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https://bu-disc.github.io/CS660/

External Sorting

Intro & 2-way external sorting

General external sorting & performance analysis

Using B^+-Trees for sorting

Why Sort?

a *classic problem* in computer science!

but also a *database specific* problem, with many use cases:

Why Sort?

a *classic problem* in computer science!

but also a *database specific* problem, with many use cases:

(i) data requested in sorted order

e.g., find students in increasing *gpa* order (using ORDER BY)

(ii) *bulk loading* B+ tree index

(iii) eliminating *duplicates* *(why?)*

(iv) summarizing groups of tuples *(what is that?)* GROUP BY!

(v) *Sort-merge* join [more about that later]

Sorting Challenges

(easy) problem:

how to sort 1GB data with 1GB memory?

(hard) problem:

how to sort 1GB data with **1MB** memory?

why not virtual memory (i.e., swapping on disk)?

Goal

minimize disk accesses when working under memory constraints

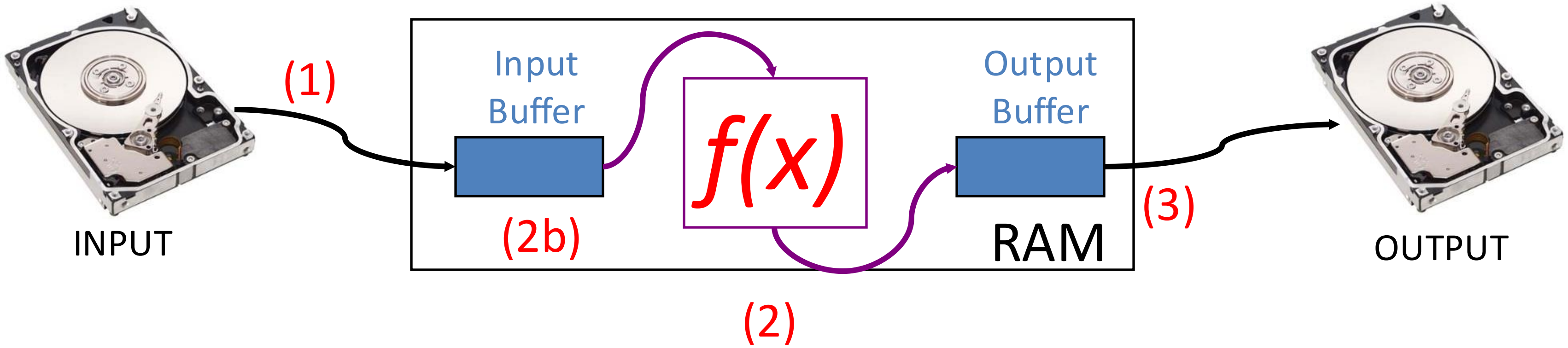
Idea

stream data, calculate *something useful*, and write back on disk

Streaming Data Through RAM

An important method for sorting & other DB operations

Compute *f(x)* for each record, write out the result



(1) Read a page (from INPUT to Input Buffer)

(3) When Output Buffer fills, write it to OUTPUT

(2) Calculate f(x) for each item (e.g., sort, (de-)compress, discard rows [selection], discard columns [projection]

(2b) When Input Buffer is consumed, read another page

What about f() being sort()?

Note that reads and writes are not (always) coordinated!

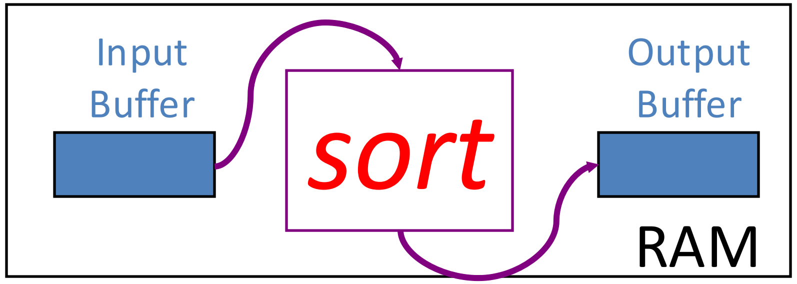
- For f() being compress(), select(), project() we may **read many pages per write**
- For f() being decompress() we may **write many pages per read**

Let’s apply this to sorting!

2-Way Sort: Requires 3 Buffers

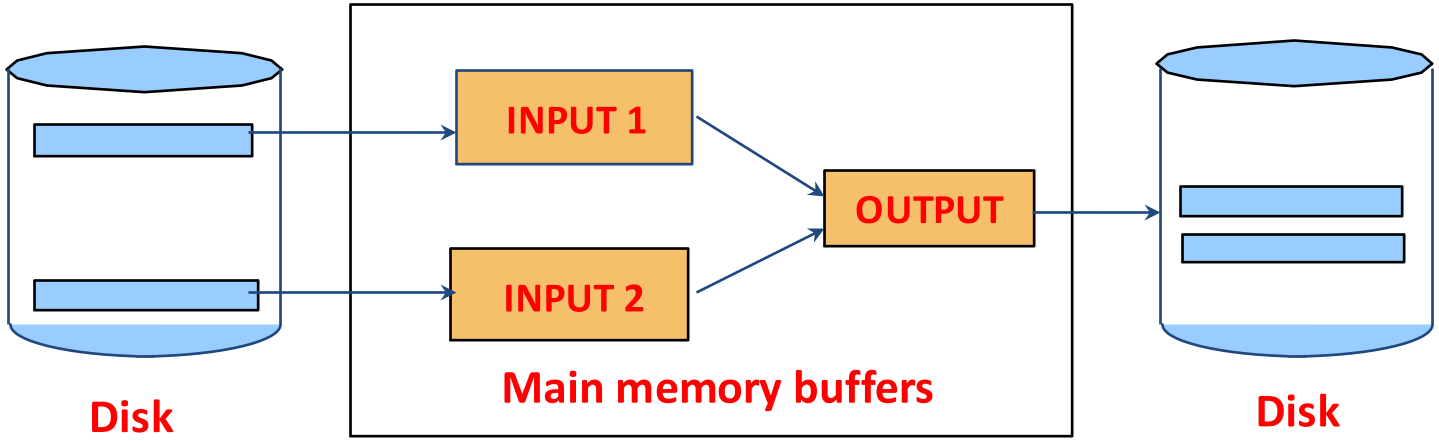
Pass 0: **Read** a page, **sort** it, **write** it.

- only one input buffer page (as in previous slide)



Pass 1, 2, 3, …, etc.:

- requires 3 buffer pages (2 input buffers)
- merge pairs of runs into runs twice as long



Two-Way External Merge Sort

Each pass we read + write each page in file.

N pages in the file =>

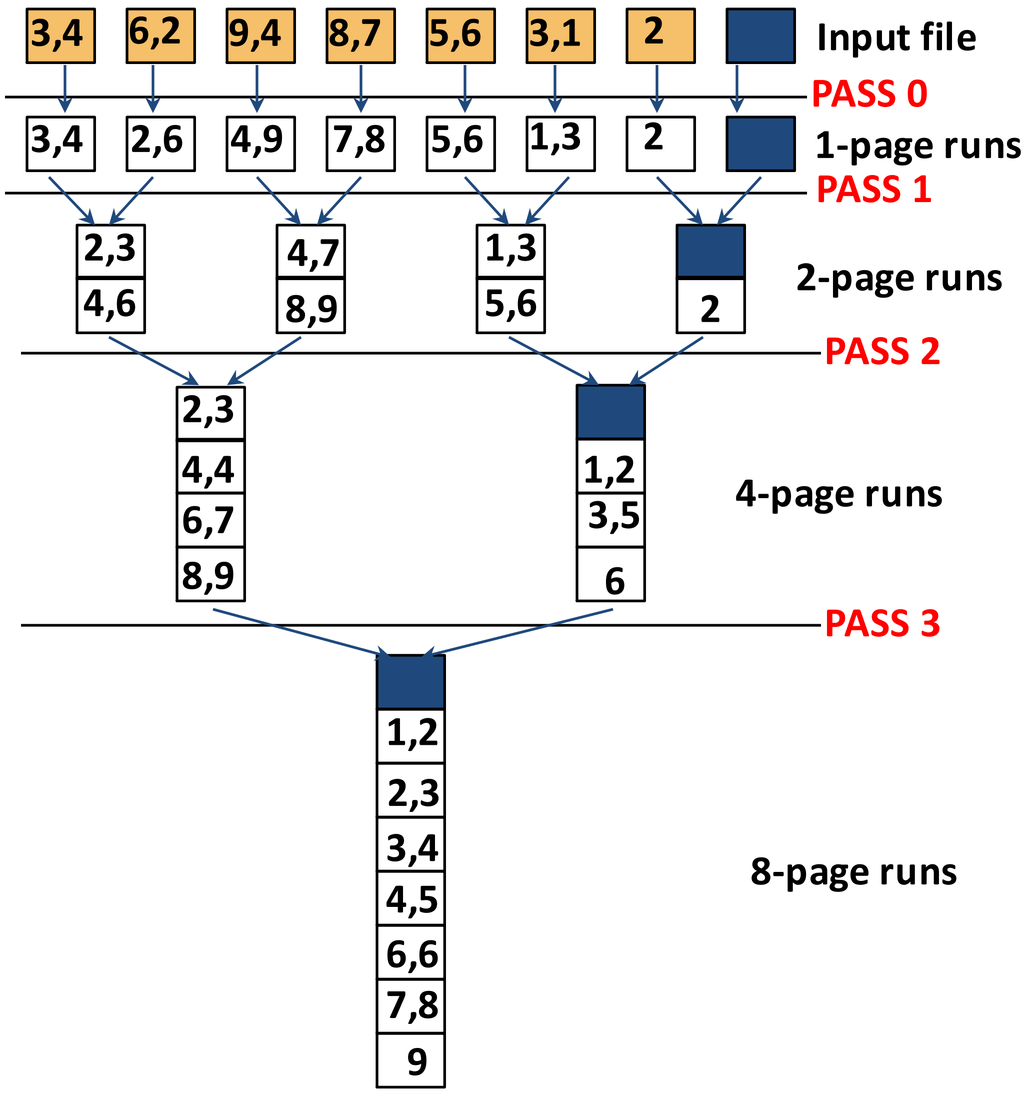
the number of passes ??

So total cost is: ??

Idea

Divide and conquer

sort sub-files and merge



Two-Way External Merge Sort

Each pass we read + write each page in file.

N pages in the file =>

the number of passes $= \lceil log_2 N \rceil + 1$

So total cost is: $2N(\lceil log_2 N \rceil + 1)$

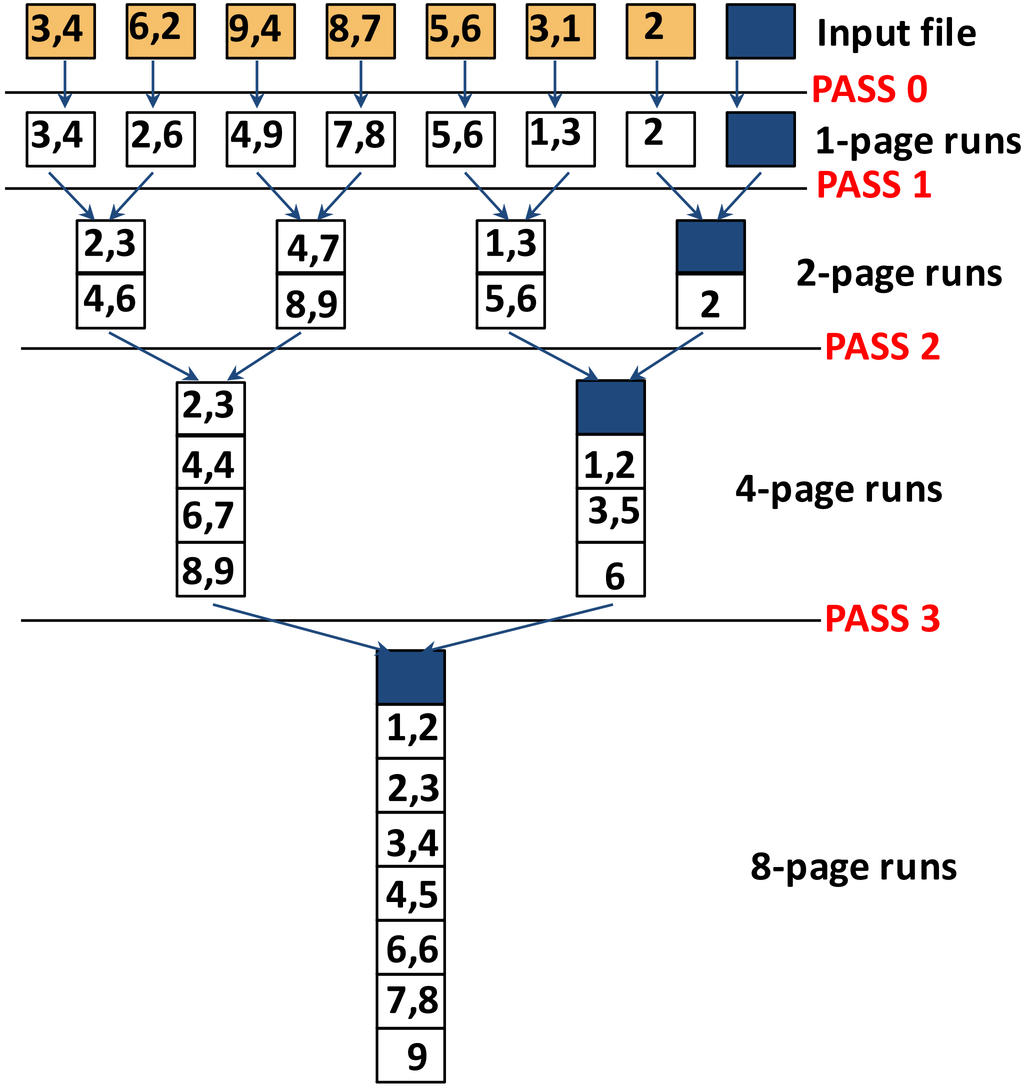
Idea

Divide and conquer

sort sub-files and merge

is this good enough?

No! why?



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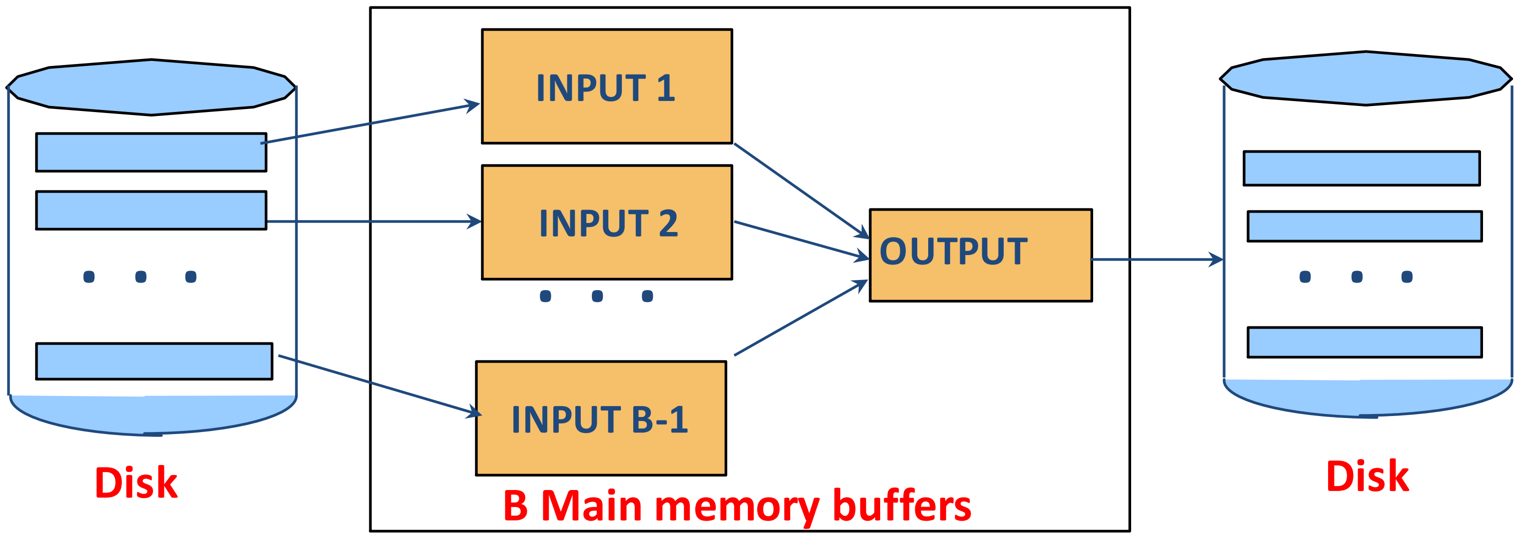
General External Merge Sort

How can we exploit more than 3 buffer pages?

To sort a file with *N* pages using *B* buffer pages:

- Pass 0: use *B* buffer pages. Produce $\lceil N/B \rceil$ sorted runs of *B* pages each.
- Pass 1, 2, …, etc.: merge *B-1* runs.



General External Merge Sort

N = 108 pages

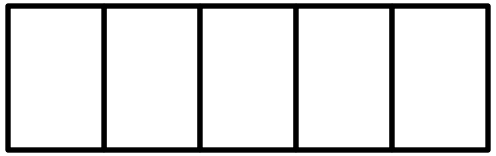
0: 5 | 5 | … $\lceil 108/5 \rceil = 22$ sorted runs of 5 pages each (last run 3 pages) | 3

1: 20 | 20 | … $\lceil 22/4 \rceil = 6$ sorted runs of $5 \cdot 4 = 20$ pages each (last run 8) | 8

2: 80 | … $\lceil 6/4 \rceil = 2$ sorted runs of $20 \cdot 4 = 20$ pages (last run 28) | 28

3: Sorted File!

B=5 buffer pages



Cost of External Merge Sort

Number of passes: $1 + \lceil log_{B-1} \lceil N/B \rceil \rceil$

Cost = $2N \cdot$ (# of passes)

to sort 108-page file with 5 buffers:

- Pass 0: $\lceil 108/5 \rceil$ = 22 sorted runs of 5 pages each (last run is only 3 pages)
- Pass 1: $\lceil 22/4 \rceil$ = 6 sorted runs of 20 pages each (last run is only 8 pages)
- Pass 2: 2 sorted runs, 80 pages and 28 pages
- Pass 3: Sorted file of 108 pages

Formula check: $1 + \lceil log_{B-1} \lceil N/B \rceil \rceil = 1 + \lceil log_4 22 \rceil$ = 1 + 3

Number of Passes of External Sort

I/O cost is 2N times number of passes: $2 \cdot \text{N} \cdot \left(1 + \lceil log_{B-1}\lceil N/B \rceil \rceil\right)$

N	B=3	B=5	B=9	B=17	B=129	B=257
100	7	4	3	2	1	1
1,000	10	5	4	3	2	2
10,000	13	7	5	4	2	2
100,000	17	9	6	5	3	3
1,000,000	20	10	7	5	3	3
10,000,000	23	12	8	6	4	3
100,000,000	26	14	9	7	4	4
1,000,000,000	30	15	10	8	5	4

In-Memory Sort Algorithm

Quicksort is fast (very fast)!!

we generate in Pass 0 N/B #runs of B pages each

can we generate longer runs?
why do we want that?

yes! Idea: maintain a current set as a heap

In-memory Heapsort

(aka “replacement sort”)

0: read in B-2 blocks

1: find the smallest record greater than the largest value to output buffer

- add it to the end of the output buffer
- fill moved record’s slot with next value from the input buffer, if empty refill input buffer

2: **else:** end run

3: goto (1)

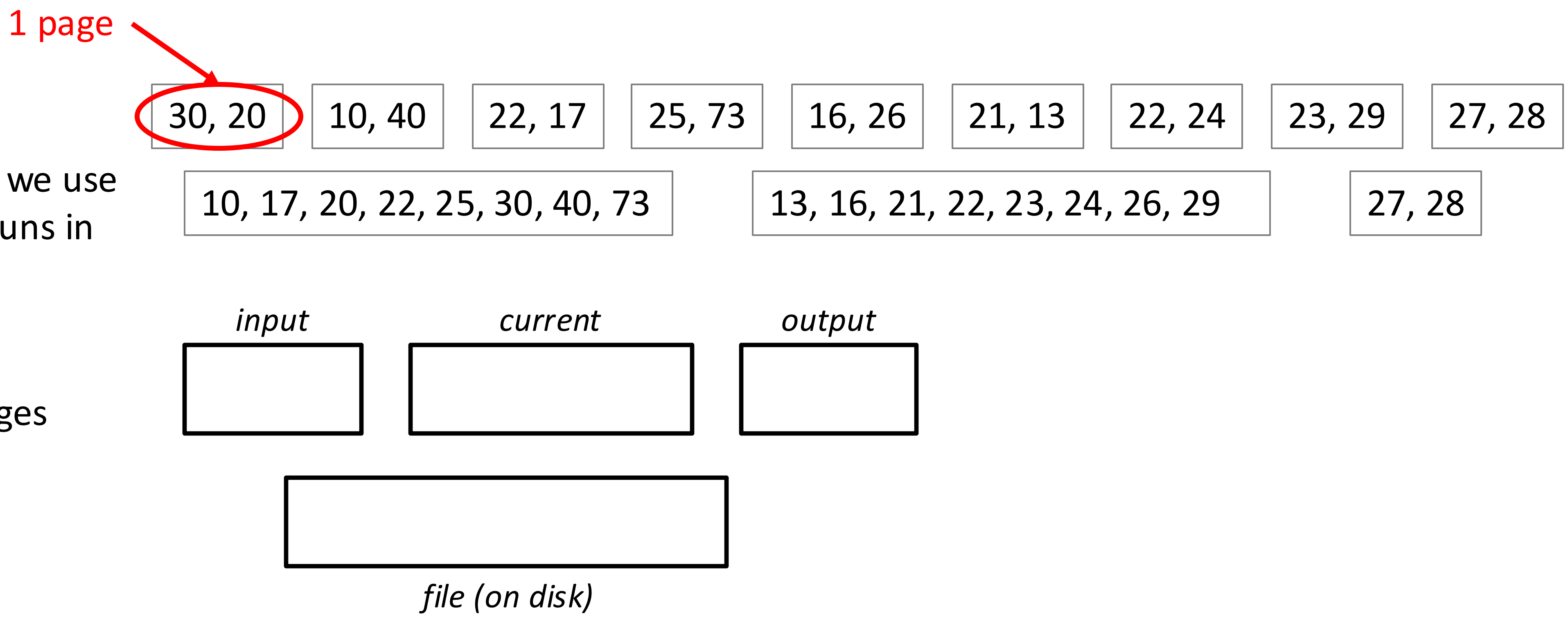
In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

up to 4 pages

Normally we use 3-pages runs in Pass 0

Heapsort 4-2=2 pages



In-memory Heapsort

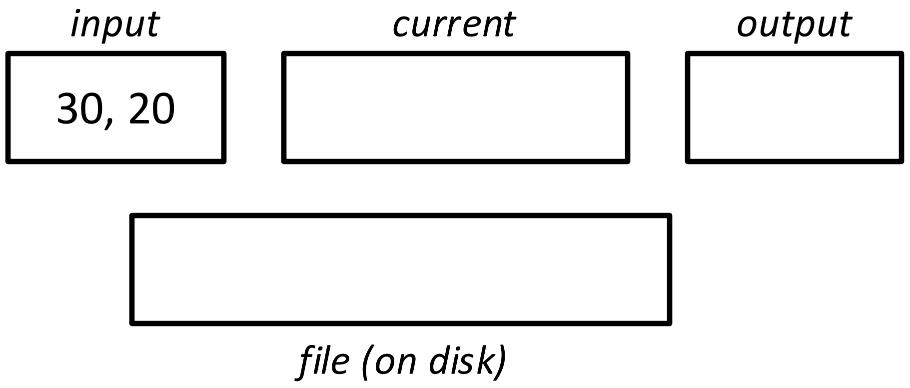
N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages



In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input   *current*   *output*

current: 20, 30

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
10, 40	20, 30	

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
	10, 20, 30, 40	

file (on disk)

In-memory Heapsort

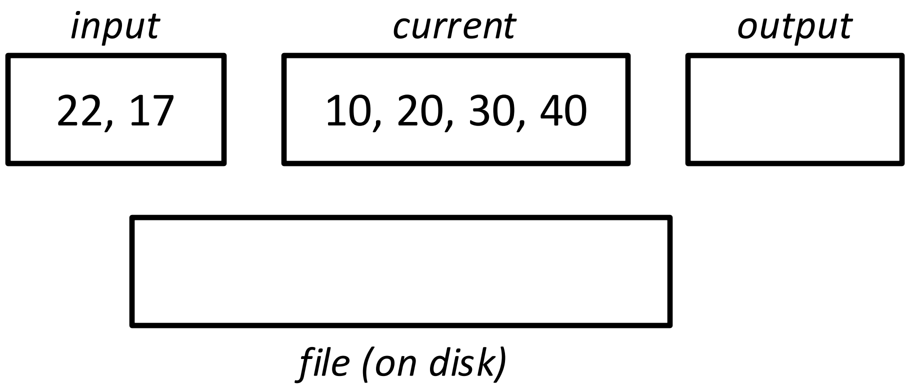
N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages



In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
22, 17	20, 30, 40	10

pop from the heap the smallest value

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input: 22

current: 17, 20, 30, 40

output: 10

update the current heap from input

file (on disk)

In-memory Heapsort

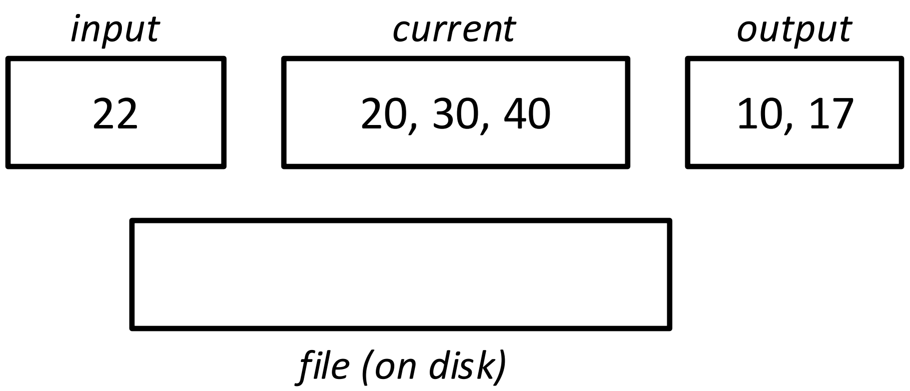
N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages



pop from the heap the smallest value

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

input | *current*: 20, 22, 30, 40 | *output*: 10, 17

update the current heap from input

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
25, 73	20, 22, 30, 40	10, 17

buffers are full

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
25, 73	20, 22, 30, 40	

flush output to disk

10, 17

file (on disk)

In-memory Heapsort

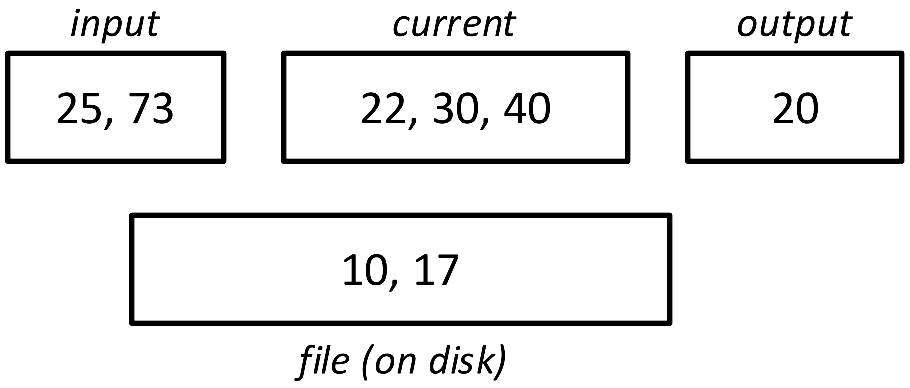
N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort
4-2=2 pages



pop heap

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
73	22, 25, 30, 40	20

update current

10, 17

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
73	25, 30, 40	20, 22

pop heap

10, 17

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
	25, 30, 40, 73	20, 22

update current from input

10, 17

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
	25, 30, 40, 73	20, 22

update current from input

10, 17

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input: 16, 26

current: 25, 30, 40, 73

output: 20, 22

buffers are full

10, 17

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
16, 26	25, 30, 40, 73	

flush output to disk

10, 17, 20, 22

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
16, 26	30, 40, 73	25

pop smallest value from heap

10, 17, 20, 22

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

	30, 20	10, 40	22, 17	25, 73	16, 26	21, 13	22, 24	23, 29	27, 28
Normally we use 3-pages runs in Pass 0	10, 17, 20, 22, 25, 30, 40, 73			13, 16, 21, 22, 23, 24, 26, 29			27, 28		

Heapsort
4-2=2 pages

input	*current*	*output*
16	26, 30, 40, 73	25

10, 17, 20, 22

file (on disk)

update current from input

16 cannot be used since 22 is on disk

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
16	30, 40, 73	25, 26

pop smallest value from heap

10, 17, 20, 22

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
16	30, 40, 73	

flush to disk

10, 17, 20, 22, 25, 26

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort
4-2=2 pages

input: 16

current: 73

output: 30, 40

pop smallest values from heap

10, 17, 20, 22, 25, 26

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
16	73	

cannot load, so flush

10, 17, 20, 22, 25, 26, 30, 40

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
	16	73

pop heap & update current

10, 17, 20, 22, 25, 26, 30, 40

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
	16	

nothing to do, flush

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input: 21, 13

current: 16

output:

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 | 10, 40 | 22, 17 | 25, 73 | 16, 26 | 21, 13 | 22, 24 | 23, 29 | 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 | 13, 16, 21, 22, 23, 24, 26, 29 | 27, 28

Heapsort 4-2=2 pages

input: (empty)

current: 13, 16, 21

output: (empty)

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 | 10, 40 | 22, 17 | 25, 73 | 16, 26 | 21, 13 | 22, 24 | 23, 29 | 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 | 13, 16, 21, 22, 23, 24, 26, 29 | 27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
22, 24	13, 16, 21	

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 | 10, 40 | 22, 17 | 25, 73 | 16, 26 | 21, 13 | 22, 24 | 23, 29 | 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 | 13, 16, 21, 22, 23, 24, 26, 29 | 27, 28

Heapsort 4-2=2 pages

input: 24

current: 13, 16, 21, 22

output: (empty)

update current

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73

13, 16, 21, 22, 23, 24, 26, 29

27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
24	16, 21, 22	13

pop smallest value from heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 | 10, 40 | 22, 17 | 25, 73 | 16, 26 | 21, 13 | 22, 24 | 23, 29 | 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 | 13, 16, 21, 22, 23, 24, 26, 29 | 27, 28

Heapsort 4-2=2 pages

input: (empty)

current: 16, 21, 22, 24

output: 13

update current

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
23, 29	16, 21, 22, 24	13

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20	10, 40	22, 17	25, 73	16, 26	21, 13	22, 24	23, 29	27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 | 13, 16, 21, 22, 23, 24, 26, 29 | 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
23, 29	21, 22, 24	13, 16

pop smallest value from heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
23, 29	21, 22, 24	

flush to new file

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
23, 29	22, 24	21

pop smallest value of heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
	22, 23, 24, 29	21

update current

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
27, 28	22, 23, 24, 29	21

update current

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
27, 28	23, 24, 29	21, 22

pop smallest value of heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
27, 28	23, 24, 29	

flush to disk

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73   13, 16, 21, 22, 23, 24, 26, 29   27, 28

Heapsort
4-2=2 pages

input	*current*	*output*
	24, 27, 28, 29	23

pop heap & update current

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 | 10, 40 | 22, 17 | 25, 73 | 16, 26 | 21, 13 | 22, 24 | 23, 29 | 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 | 13, 16, 21, 22, 23, 24, 26, 29 | 27, 28

Heapsort 4-2=2 pages

input	*current*	*output*
	27, 28, 29	23, 24

pop heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

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Heapsort 4-2=2 pages

input	*current*	*output*
	27, 28, 29	

flush to disk

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22, 23, 24

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

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Heapsort 4-2=2 pages

input	*current*	*output*
	29	27, 28

pop heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22, 23, 24

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20   10, 40   22, 17   25, 73   16, 26   21, 13   22, 24   23, 29   27, 28

Normally we use 3-pages runs in Pass 0

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Heapsort 4-2=2 pages

input	*current*	*output*
	29	

flush to disk

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22, 23, 24, 27, 28

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

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Heapsort 4-2=2 pages

input	*current*	*output*
		29

pop heap

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

13, 16, 21, 22, 23, 24, 27, 28

file (on disk)

In-memory Heapsort

N = 9 pages (file), B = 4 pages (buffers)

30, 20 10, 40 22, 17 25, 73 16, 26 21, 13 22, 24 23, 29 27, 28

Normally we use 3-pages runs in Pass 0

10, 17, 20, 22, 25, 30, 40, 73 13, 16, 21, 22, 23, 24, 26, 29 27, 28

Heapsort 4-2=2 pages

input *current* *output*

flush to disk

10, 17, 20, 22, 25, 26, 30, 40, 73

file (on disk)

only 2 (longer) sorted runs!

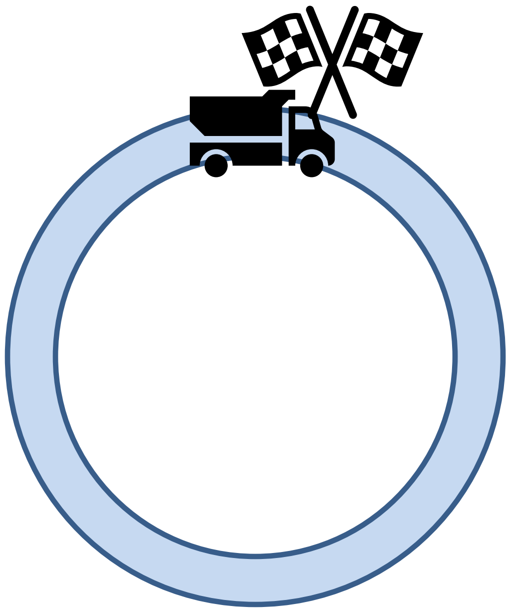
13, 16, 21, 22, 23, 24, 27, 28, 29

file (on disk)

More on Heapsort

Fact: average length of a run in heapsort is *2*B*

<u>The snowplow analogy</u>



(1) Imagine a snowplow moving around a circular track with a steady rate of snow fall.

(2) At any instant, there is a certain amount of snow S on the track. Some falling snow comes in front of the plow, some behind.

(3) During the next revolution of the plow, all of this is removed, plus 1/2 of what falls during that revolution.

(4) Thus, the plow removes 2S amount of snow.

More on Heapsort

Fact: average length of a run in heapsort is *2*B*

Worst-Case:

- What is min length of a run? B-2
- How does this arise? when the file is reversely sorted

Best-Case:

- What is max length of a run? the entire file
- How does this arise? when the file is sorted

Quicksort is faster, but ... longer runs often means fewer passes!

External Merge Sort Summary

unsorted file of N pages

0: B | B | … | B

$\lceil N/B \rceil$ sorted runs of B pages each (or, fewer of $2(B-2)$ each)

1: $B(B-1)$ | $B(B-1)$ | … | $B(B-1)$

$\frac{\lceil N/B \rceil}{B-1}$ sorted runs of $B(B-1)$ pages each

2: $B(B-1)^2$ | $B(B-1)^2$ | … | $B(B-1)^2$

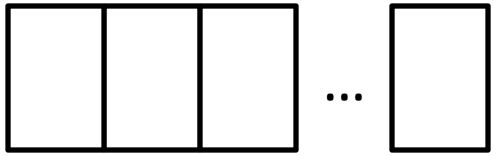
$\frac{\lceil N/B \rceil}{(B-1)^2}$ sorted runs of $B(B-1)^2$ pages each

…

$\log_{B-1}\left(\left\lceil \frac{N}{B} \right\rceil\right)$: $\frac{\lceil N/B \rceil}{(B-1)^{\log_{B-1}(\lceil N/B \rceil)}} = 1$ sorted run! of $B \cdot (B-1)^{\log_{B-1}(\lceil N/B \rceil)} = B \cdot \left\lceil \frac{N}{B} \right\rceil = N$ pages

total #I/O: $2 \cdot N \cdot (1 + \lceil \log_{B-1}(\lceil N/B \rceil) \rceil)$

B buffer pages:



I/O for External Merge Sort

- Do I/O a page at a time
  - Not one I/O per record
- In fact, read a ***block*** *(chunk)* of pages sequentially!
- Suggests we should make each buffer (input/output) be a *block* of pages (e.g., b=32 pages).
  - But this will reduce fan-in during merge passes!
  - In practice, most files still sorted in 2-3 passes.

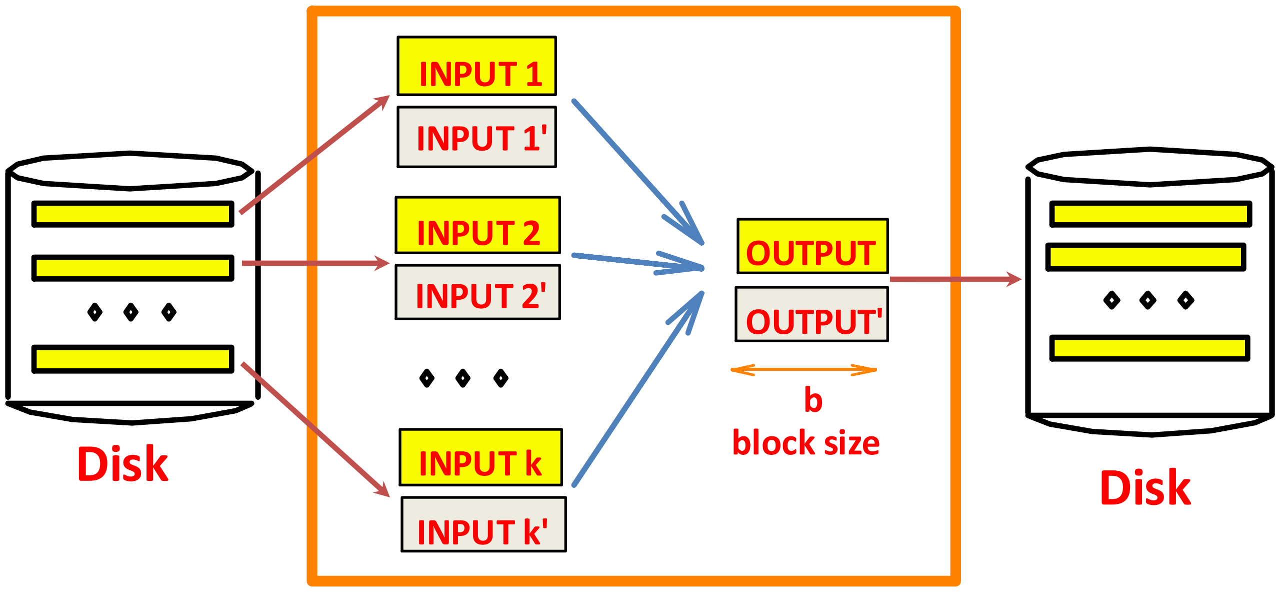
total #I/O: $2 \cdot N \cdot \left(1 + \left\lceil \log_{\lfloor B/b \rfloor}(\lceil N/B \rceil) \right\rceil\right)$

Fanout reduced from $B - 1$ to $\lfloor B/b \rfloor$, but there is locality and sequential reads

Double Buffering

To reduce wait time for I/O request to complete, can *prefetch* into "shadow block".

- Potentially, more passes; in practice, most files *still* sorted in 2-3 passes.



B main memory buffers, k-way merge

Sorting Records!

Sorting has become a blood sport!

- Parallel sorting is the name of the game ...

Minute Sort: how many 100-byte records can you sort in a minute? 37 TB

Cloud Sort: what is the cost for sorting 100TB of data? $97

Joule Sort: how many joules needed for 1TB of data? 63KJ

See http://sortbenchmark.org/

External Sorting

Intro & 2-way external sorting

General external sorting & performance analysis

Using B^+-Trees for sorting

Using B+ Trees for Sorting

Scenario: Table to be sorted has B+ tree index on sorting column(s).

Idea: Can retrieve records in order by traversing leaf pages.

Is this a good idea?

Cases to consider:

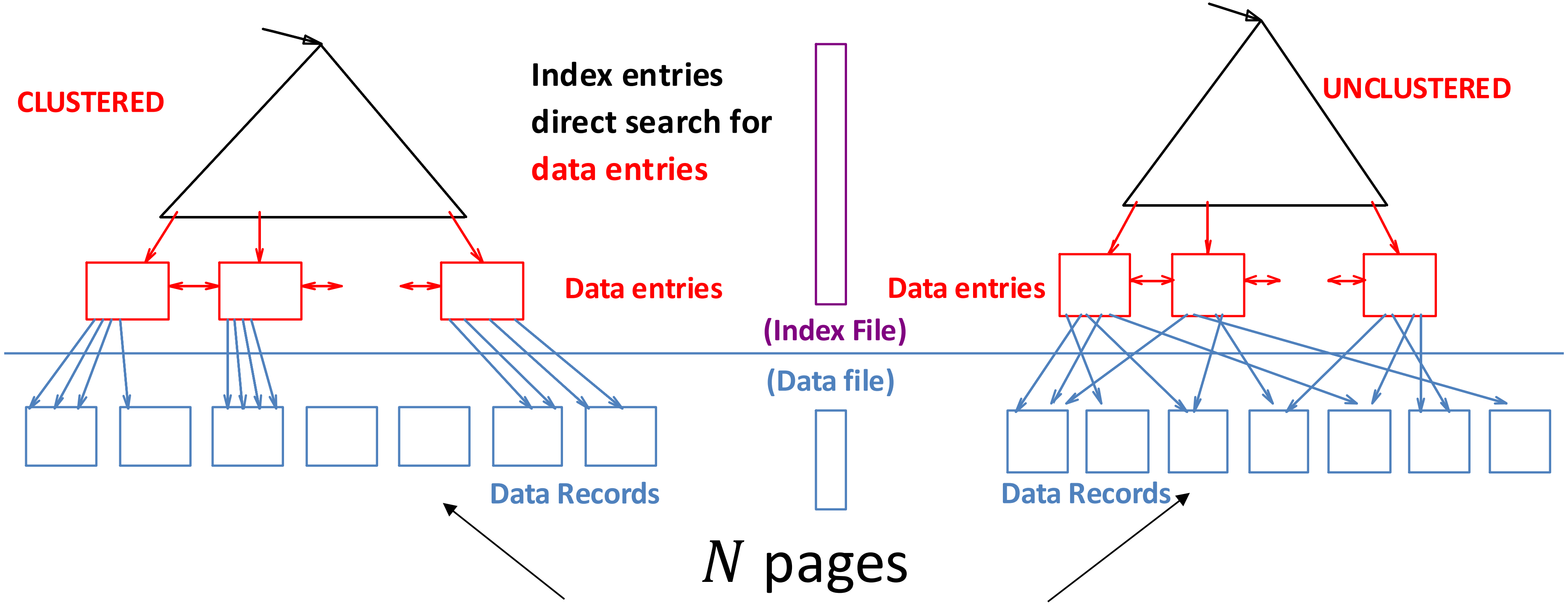
- B+ tree is clustered
- B+ tree is not clustered

Reminder: Clustered vs. Unclustered

Cost of retrieving records found in range scan:

Clustered I/O cost = one I/O per page of matching tuples (N page accesses)

Unclustered I/O cost ≈ one I/O per matching tuple ($N \cdot p$ page accesses)



N pages

p tuples per page

Using B+ Trees for Sorting

Scenario: Table to be sorted has B+ tree index on sorting column(s).

Idea: Can retrieve records in order by traversing leaf pages.

Is this a good idea?

Cases to consider:

- B+ tree is clustered   ***Good idea → one I/O per page!***
- B+ tree is not clustered

Using B+ Trees for Sorting

Scenario: Table to be sorted has B+ tree index on sorting column(s).

Idea: Can retrieve records in order by traversing leaf pages.

Is this a good idea?

Cases to consider:

- B+ tree is clustered — ***Good idea → one I/O per page!***
- B+ tree is not clustered — ***Could be a very bad idea → one I/O per tuple!***

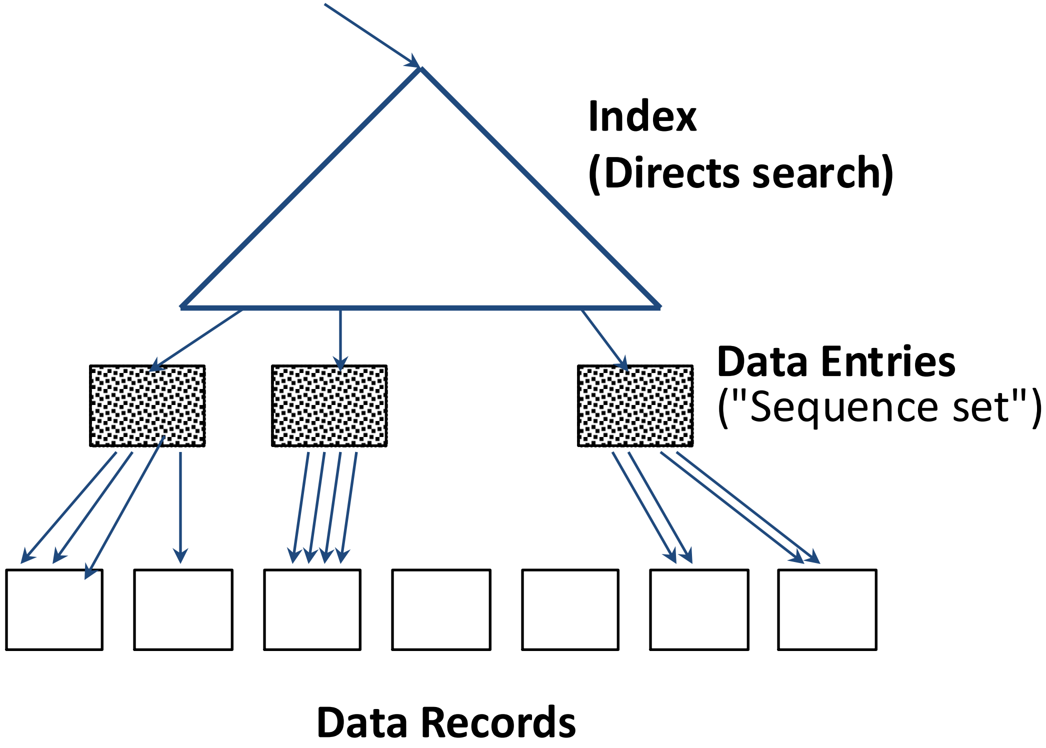
Clustered B+ Tree Used for Sorting

Cost: root to the left-most leaf, then retrieve all leaf pages (Alternative 1)

If Alternative 2 is used?

Additional cost of retrieving data entries ($^N/_p$ pages)

BUT, still each data page is fetched just once (N pages)



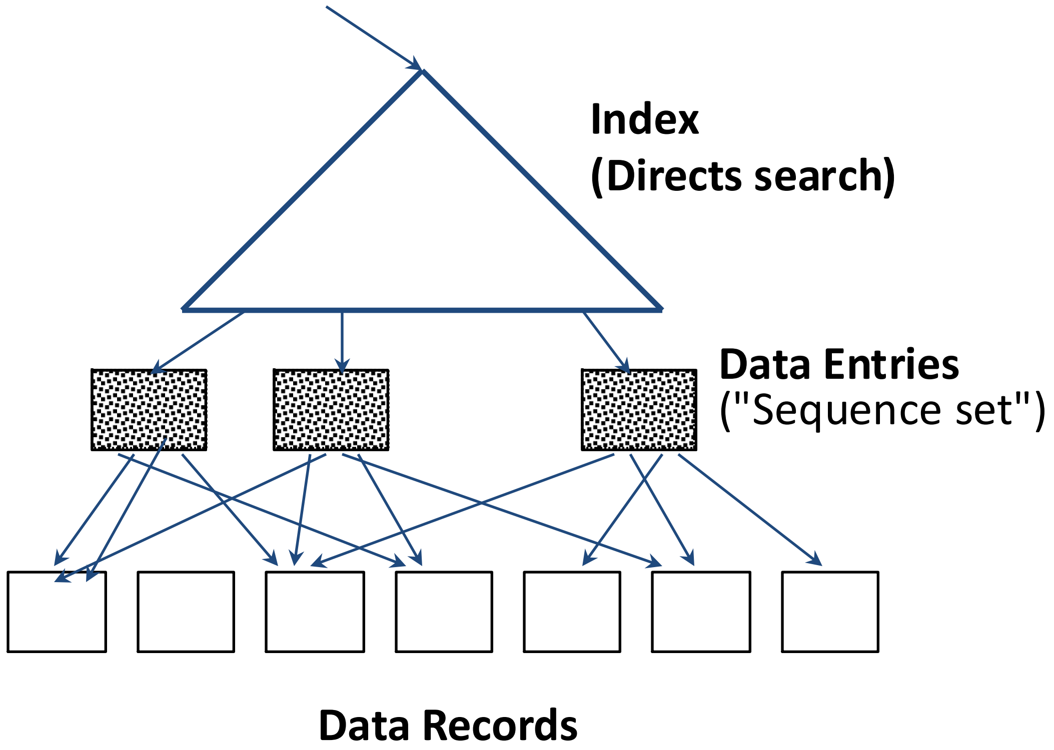
 Always better than external sorting!

 Approximate $(1 + {}^1/_p)N$ ***with*** N

Unclustered B+ Tree Used for Sorting

Alternative (2) for data entries; each data entry contains *rid* of a data record. In general, one I/O per data record!

Retrieve data entries ($^N/_p$ pages)
and
each data page p times
$(\mathrm{p} \cdot N$ page accesses)



□ ***Approximate*** $(p + {}^1/_p)N$ ***with*** $p \cdot N$

External Sorting vs. Unclustered Index

$p \cdot N$

if $B \geq N$ then only quick sort! read and write (2N)

N	Sorting	p=1	p=10	p=100
100	200	100	1,000	10,000
1,000	2,000	1,000	10,000	100,000
10,000	40,000	10,000	100,000	1,000,000
100,000	600,000	100,000	1,000,000	10,000,000
1,000,000	8,000,000	1,000,000	10,000,000	100,000,000
10,000,000	80,000,000	10,000,000	100,000,000	1,000,000,000

more realistic case

$$2 \cdot N \cdot (1 + \lceil \log_{\lfloor B/b \rfloor}(\lceil N/B \rceil) \rceil)$$

Special case, that the tree is always behaving like a clustered tree

- ***p*: # of records per page**
- ***B=1000 pages, b=32 for sorting***
- ***p=100 is the more realistic value.***

Summary

External sorting is used for many different operations in DBs

External merge sort minimizes disk I/O cost:

- Pass 0: Produces sorted ***runs*** of size ***B*** (# buffer pages). Later passes: ***merge*** runs.
- # of runs merged at a time depends on ***B***, and ***block size***.
- Larger block size means less I/O cost per page.
- Larger block size means fewer runs merged.
- In practice, # of passes rarely more than 2 or 3.

Summary, cont.

Choice of internal sort algorithm may matter:

- Quicksort: Quick!
- Heap/tournament sort: slower (2x), longer runs

The best sorts are wildly fast:

- Despite 40+ years of research, still improving!

Clustered B^+ tree is good for sorting

Unclustered tree is usually very bad