CS660: Intro to Database Systems

Class 7: Tree-Structured Indexing

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https://bu-disc.github.io/CS660/

Tree-structured indexing

Intro & B$^+$-Tree

Insert into a B$^+$-Tree

Delete from a B$^+$-Tree

Prefix Key Compression & Bulk Loading

Introduction

Recall: 3 alternatives for data entries k*:

- <**k,** entire data record>
- <**k**, rid of data record with search key value **k**>
- <**k**, list of rids of data records with search key **k**>

Choice is orthogonal to the *indexing technique* used to locate data entries k*.

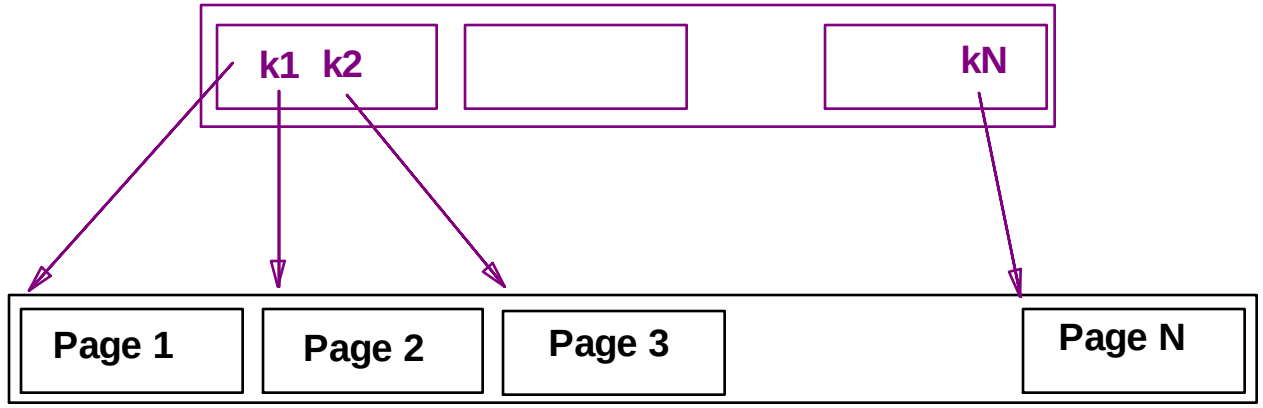
Tree-structured indexing techniques support both *range searches* and *equality searches*.

Range Searches

"Find all students with gpa > 3.0"

- If data is in sorted file, do **binary search** to find first such student, then **scan** to find others.
- Cost of **maintaining sorted file** + **performing binary search** in a database can be quite high.

Simple idea: Create an "index" file.



Index File

what can we do?

Data File

** Can do binary search on a (smaller) index file!*

B+ Tree: The Most Widely-Used Index

Insert/delete at $log_F(N)$ cost; keep tree *height-balanced*.

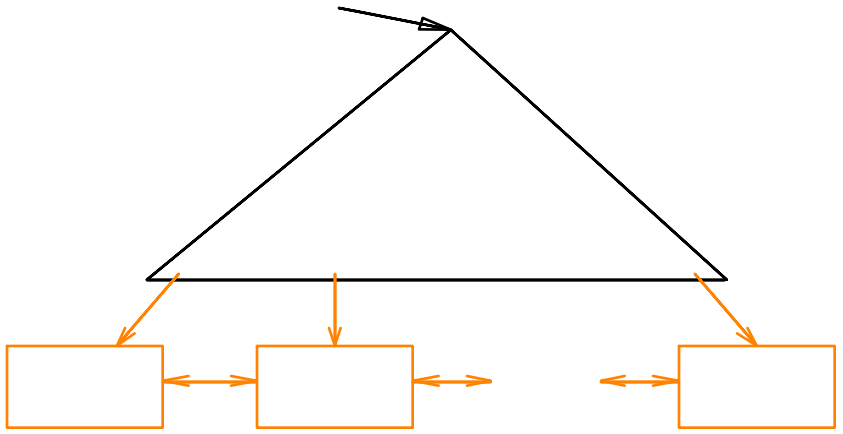
(F = fanout, N = # leaf pages)

Minimum 50% occupancy (except for root).

Each node contains $d \leq m \leq 2d$ entries. "d" is called the *order* of the tree.

Supports equality and range-searches efficiently.

All searches go from root to leaves, in a dynamic structure.



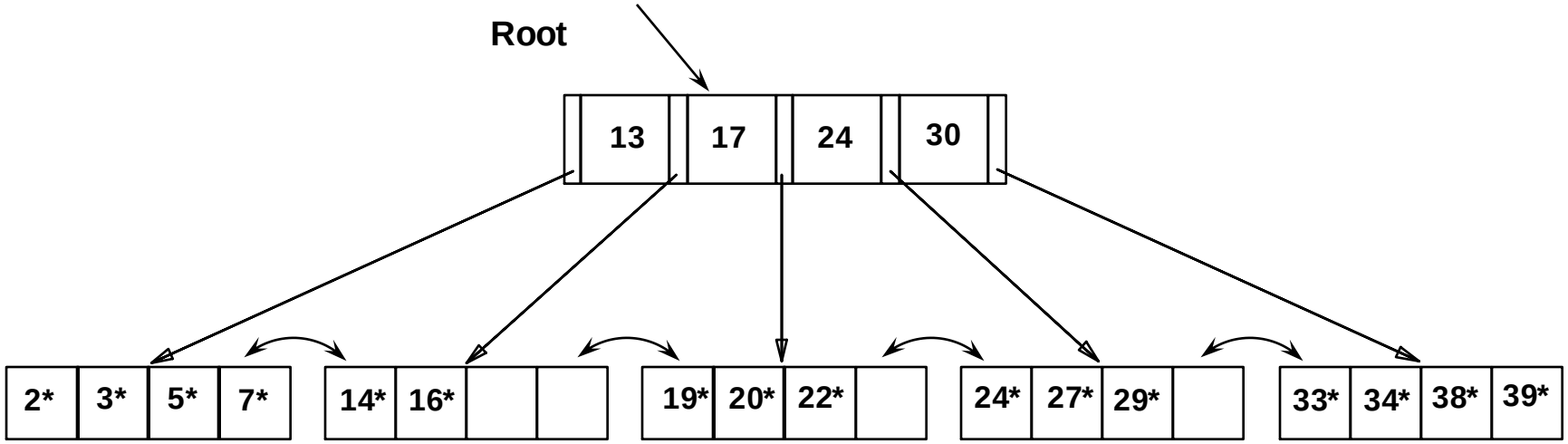
Index Entries
(Direct search)

Data Entries

Example B+ Tree

Search begins at root, and key comparisons direct it to a leaf.

Search for 5*, 15*, all data entries >= 24* ...



** Based on the search for 15*, we know it is not in the tree!*

B+ Trees in Practice (cool facts!)

Typical order: 100. Typical fill-factor: 67%.

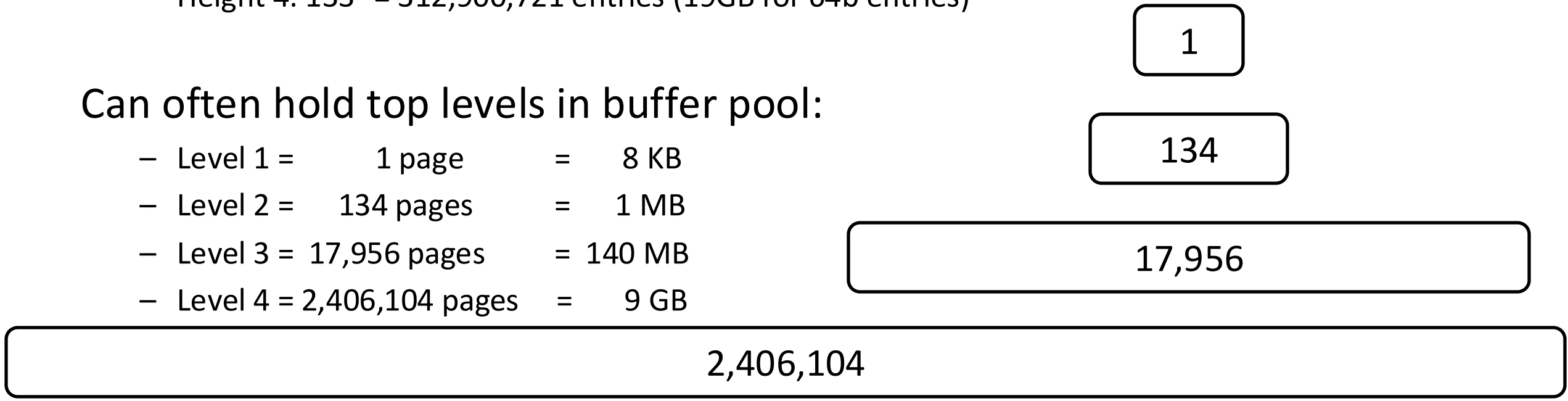
- average fanout = $2 \cdot 100 \cdot 0.67 = 134$

Typical capacities:

- Height 3: 133^3 = 2,406,104 entries
- Height 4: 133^4 = 312,900,721 entries (19GB for 64b entries)

Can often hold top levels in buffer pool:

- Level 1 = 1 page = 8 KB
- Level 2 = 134 pages = 1 MB
- Level 3 = 17,956 pages = 140 MB
- Level 4 = 2,406,104 pages = 9 GB



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Inserting a Data Entry into a B+ Tree

Find correct leaf *L.*

Put data entry onto *L*.

- If *L* has enough space, *done*!
- Else, must *split* *L (into L and a new node L2)*

  Redistribute entries evenly, **copy up** middle key.

  Insert index entry pointing to *L2* **into parent** of *L*.

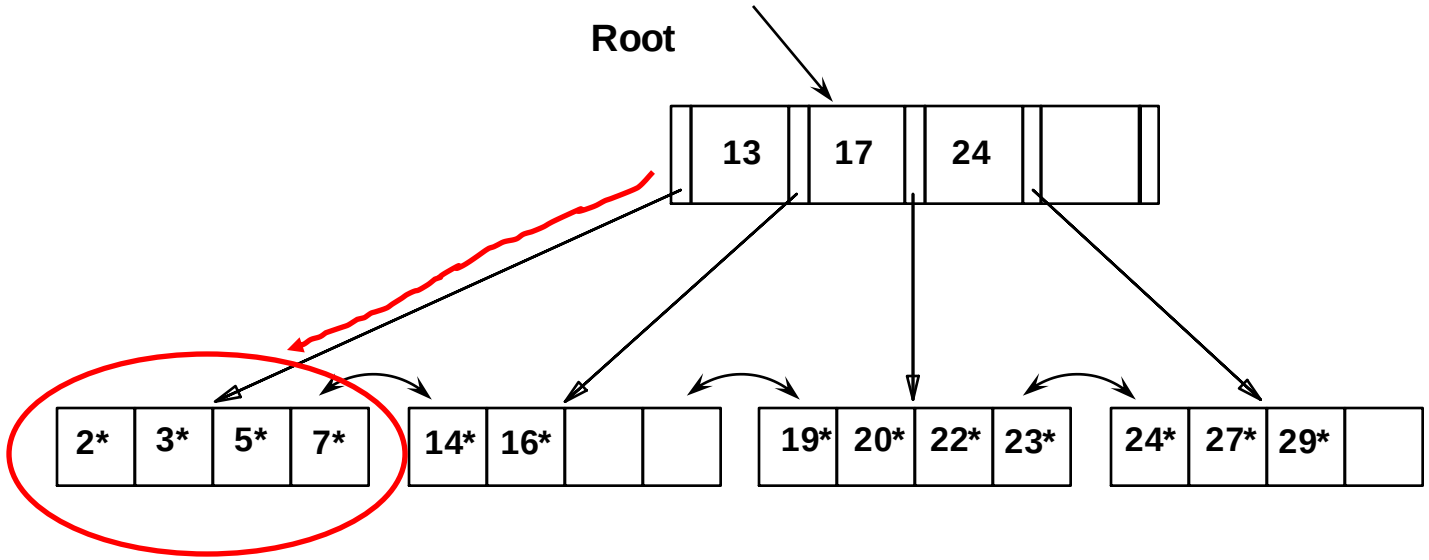
This can happen recursively

- To split index node, redistribute entries evenly, but **push up** middle key. (Contrast with leaf splits.)

Splits "grow" tree; root split increases height.

- Tree growth: gets *wider* or *one level taller at top.*

Example B+ Tree - Inserting 8*



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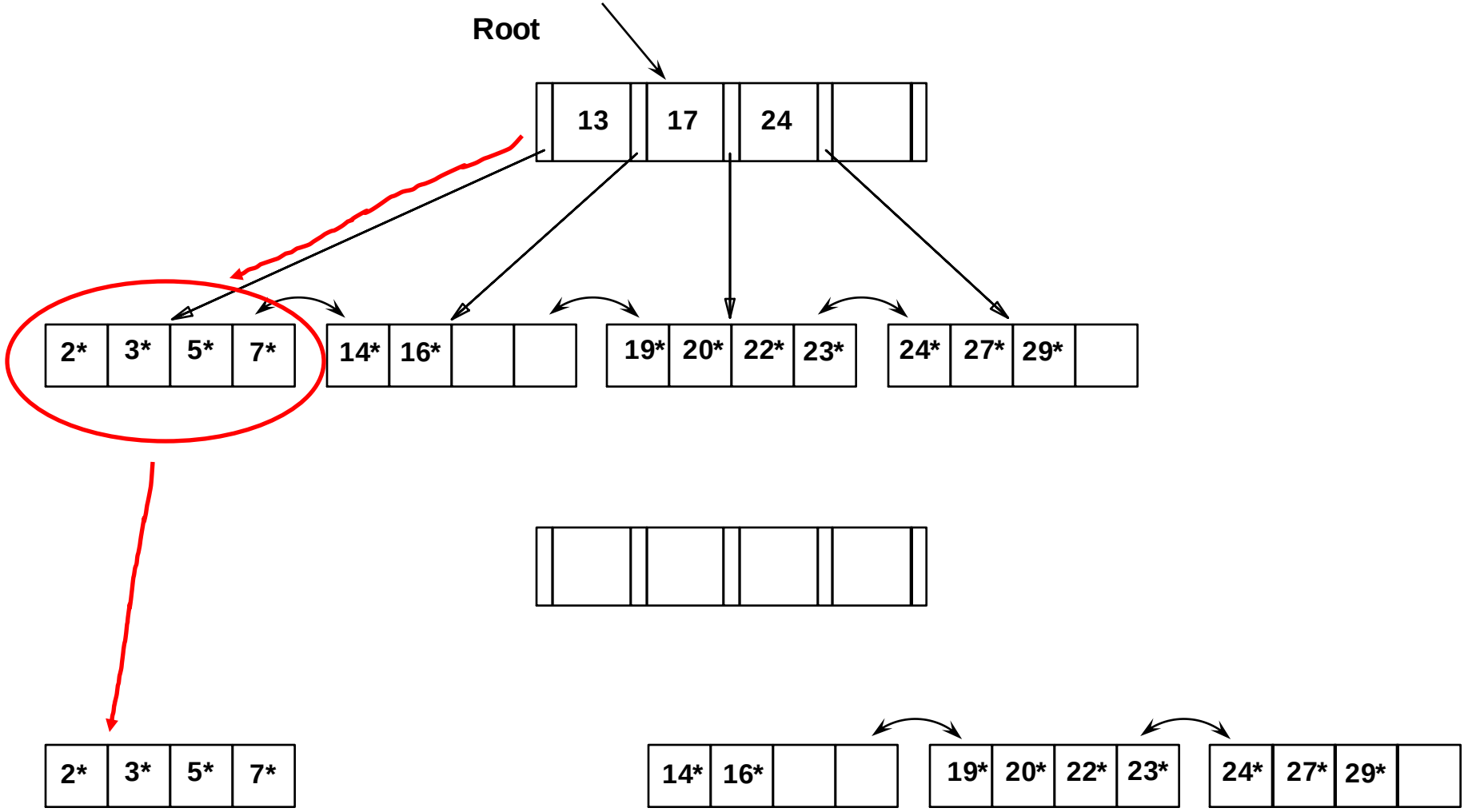

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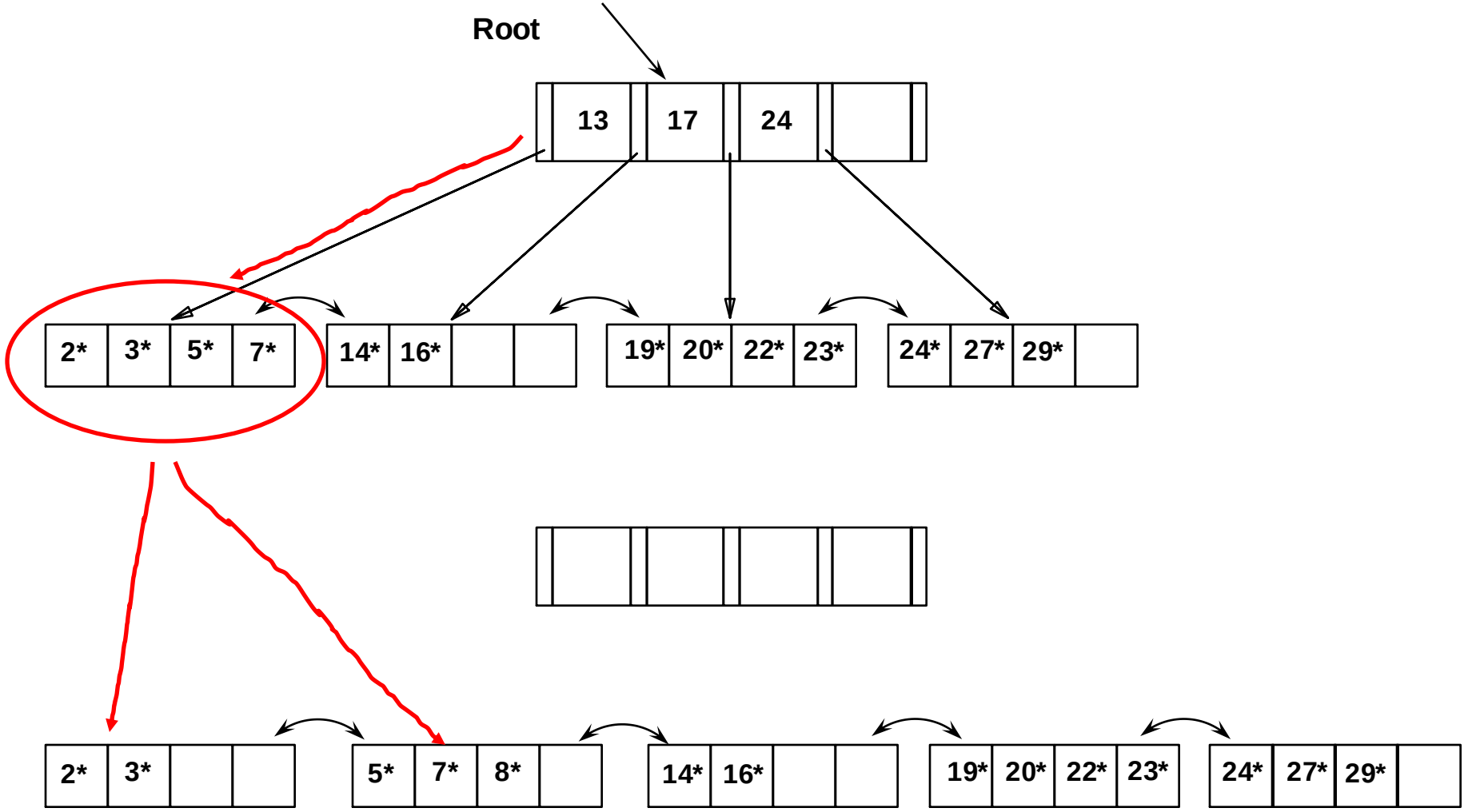

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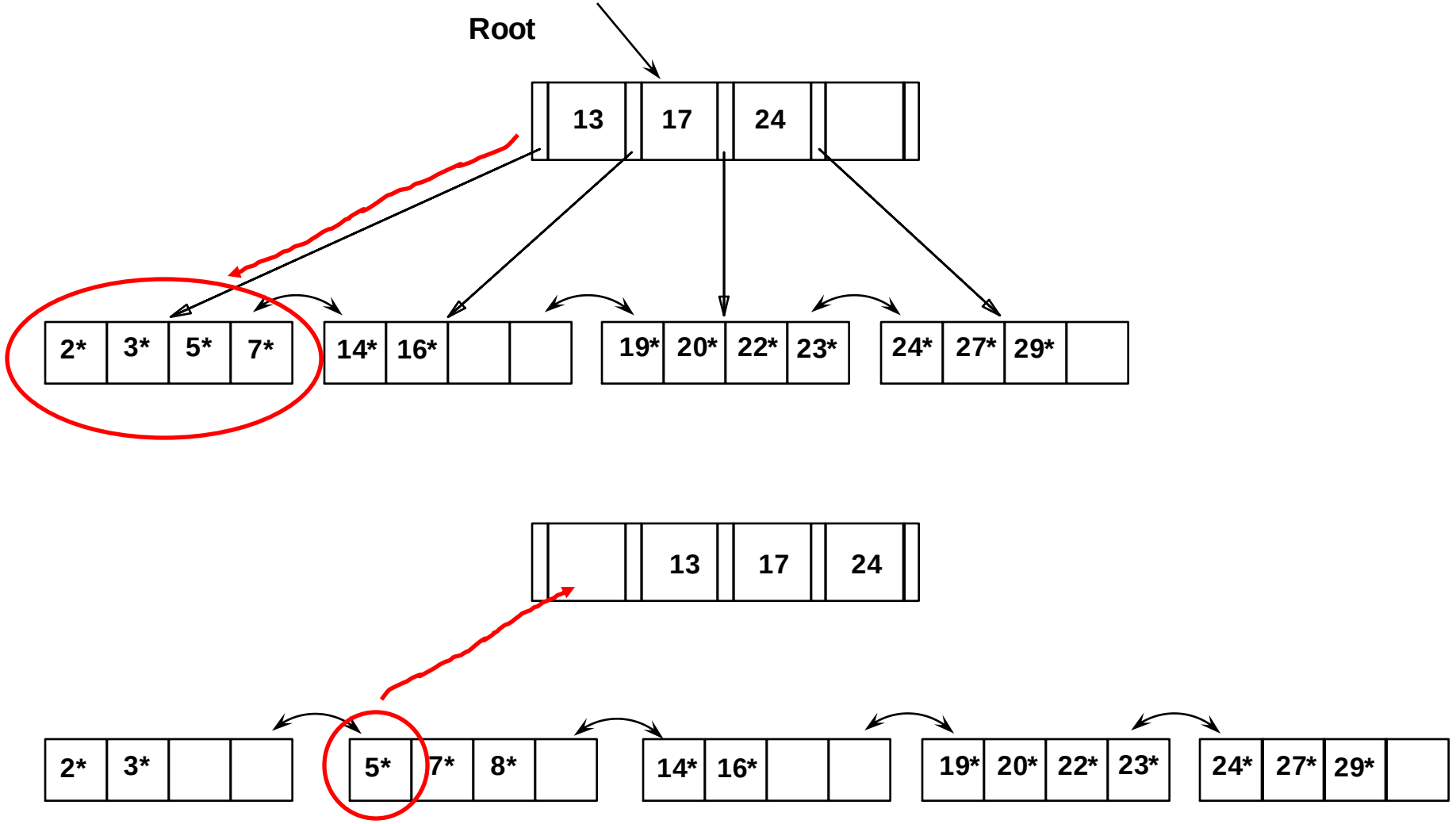

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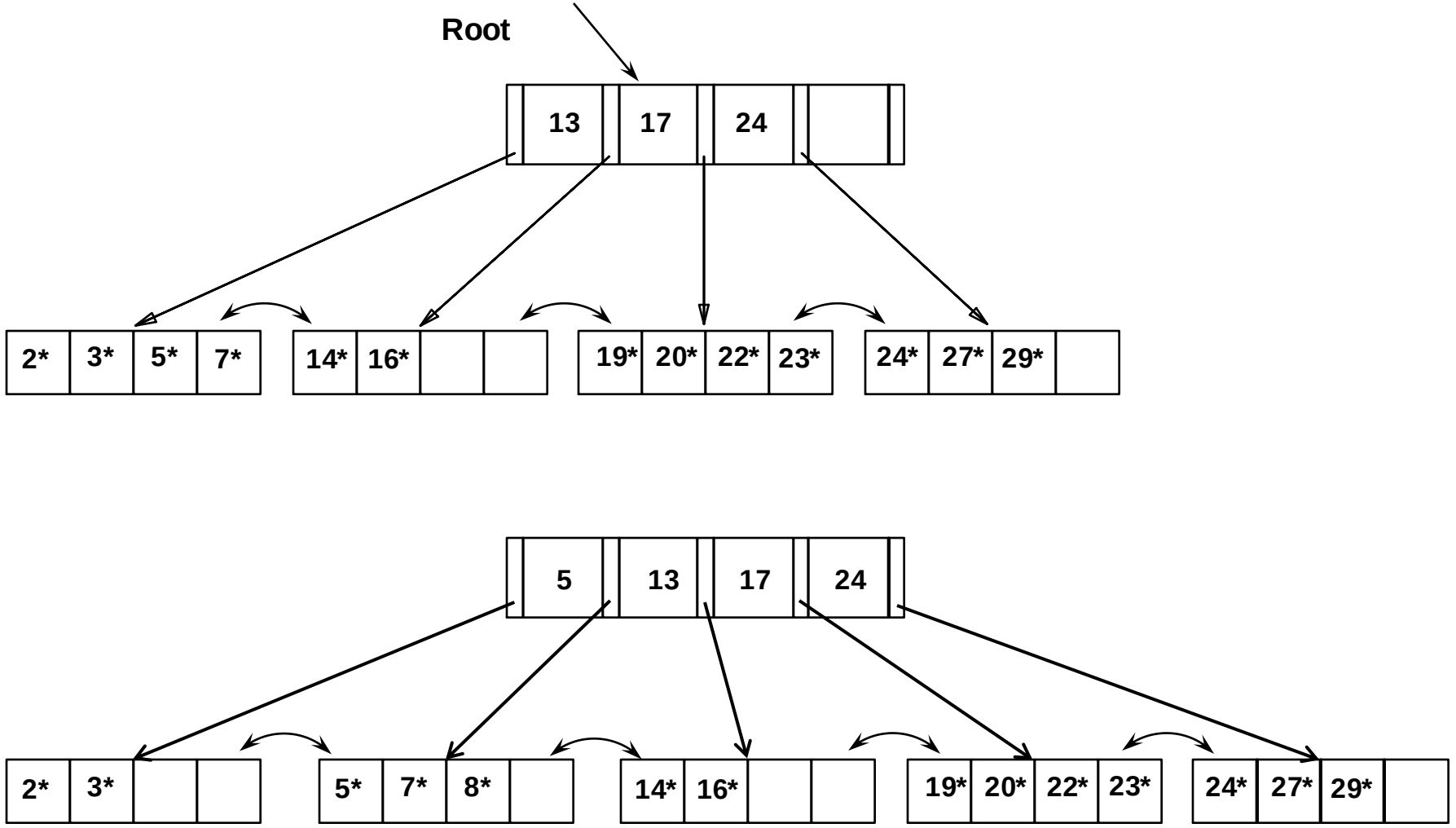
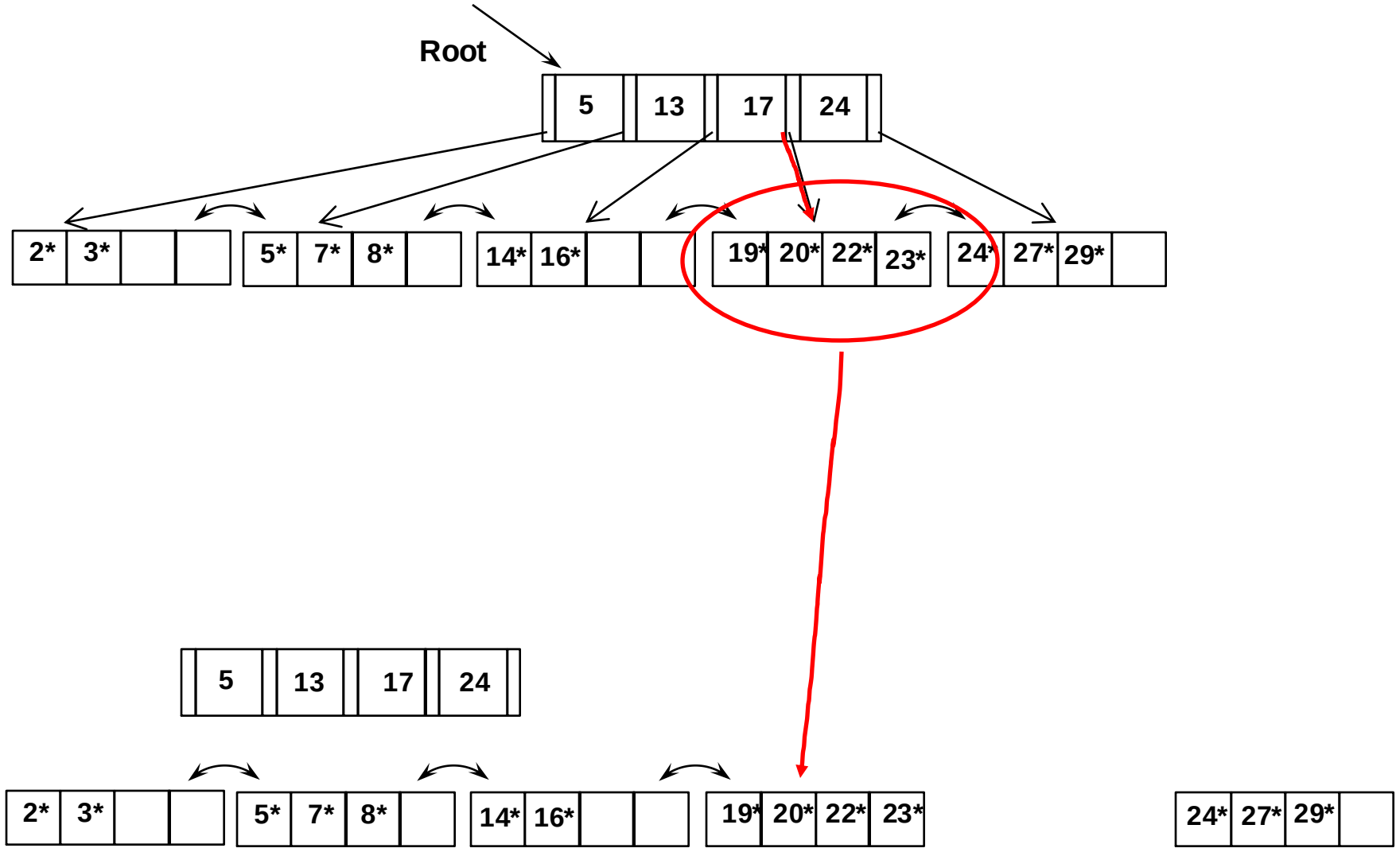


Example B+ Tree - Inserting 21*



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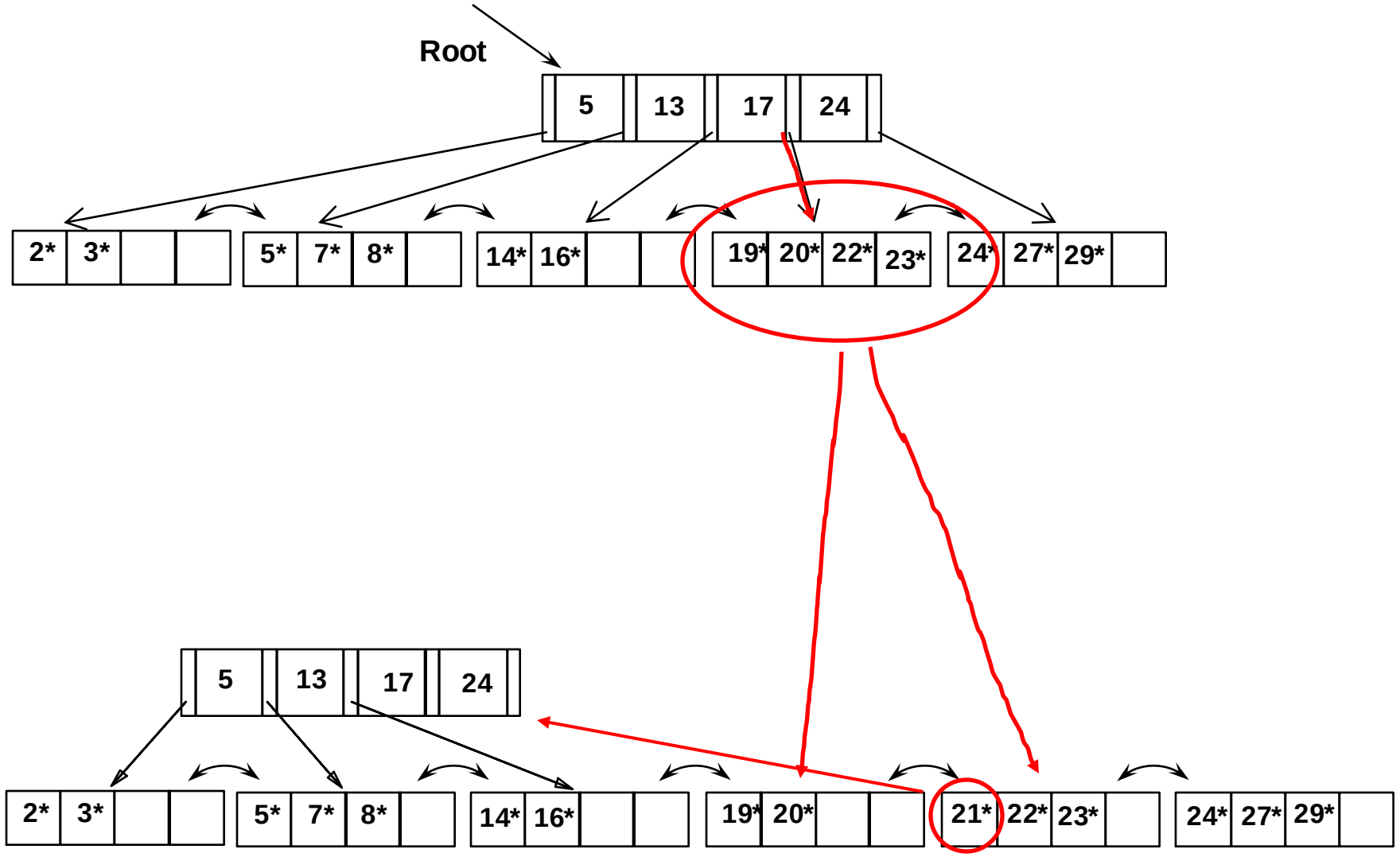

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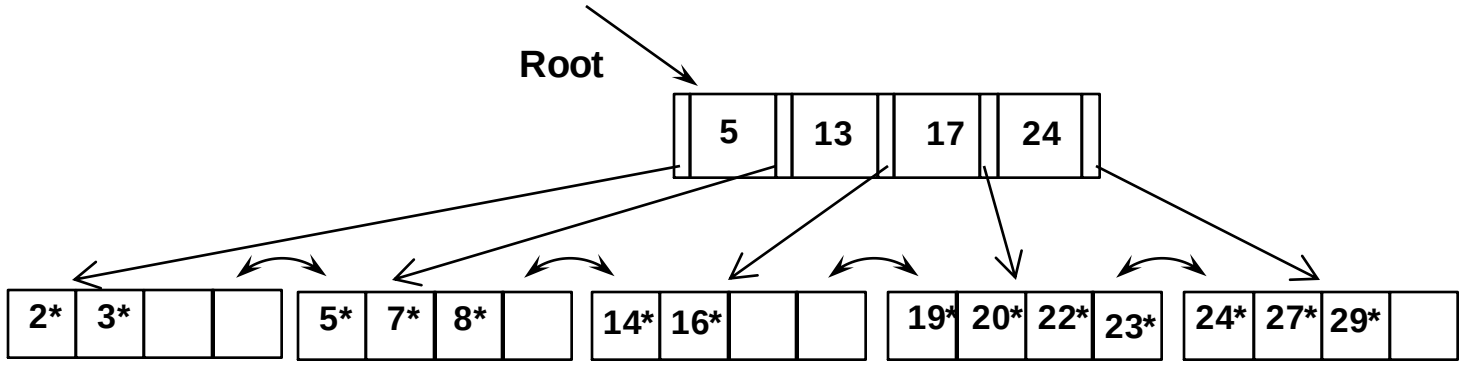

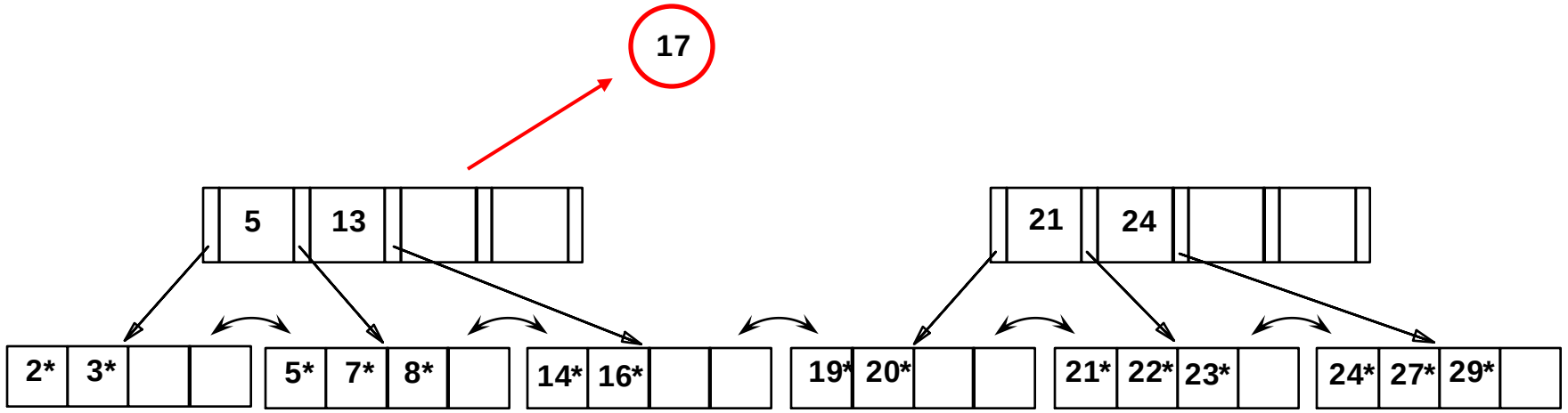

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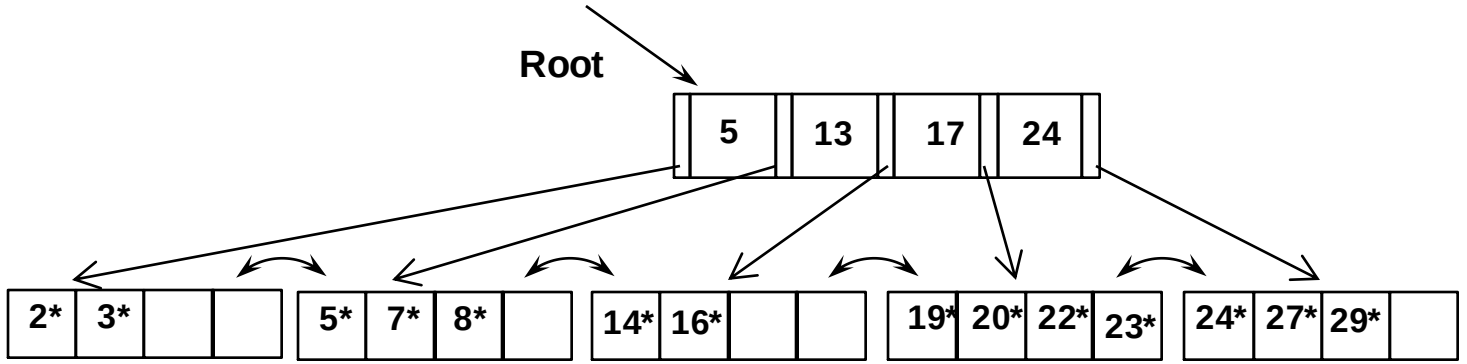

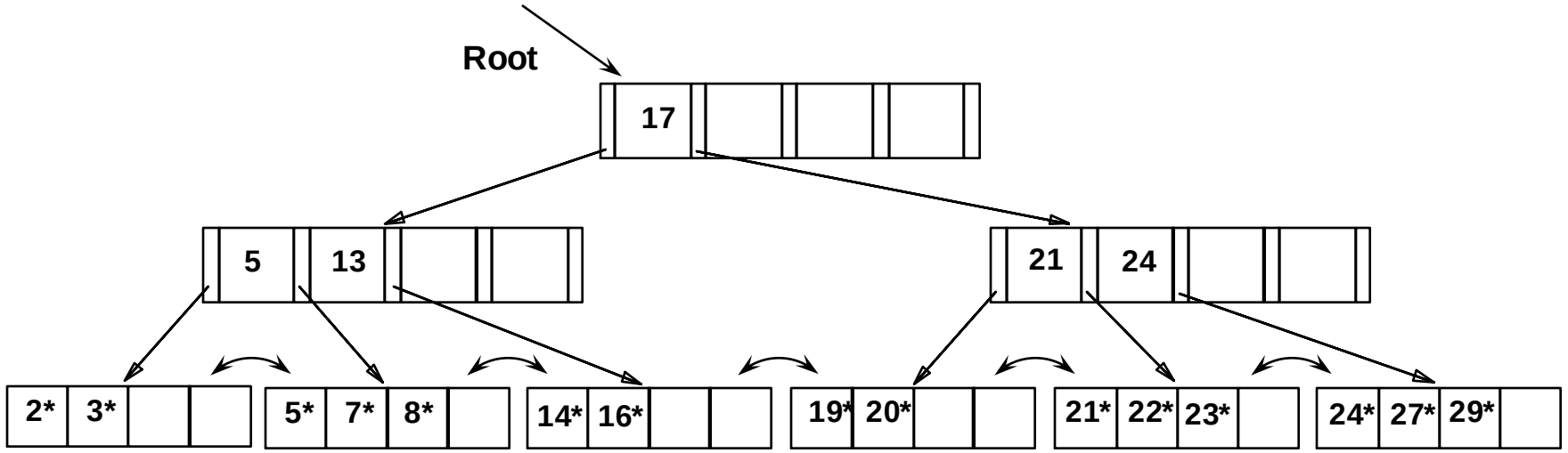

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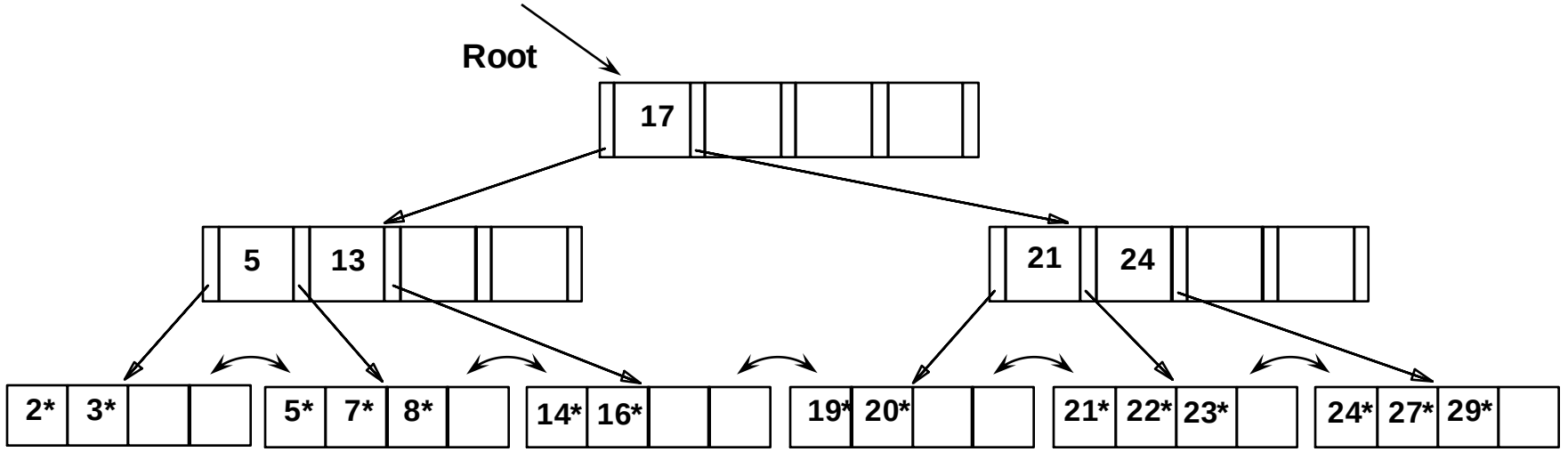


Notice that root was split, leading to increase in height.

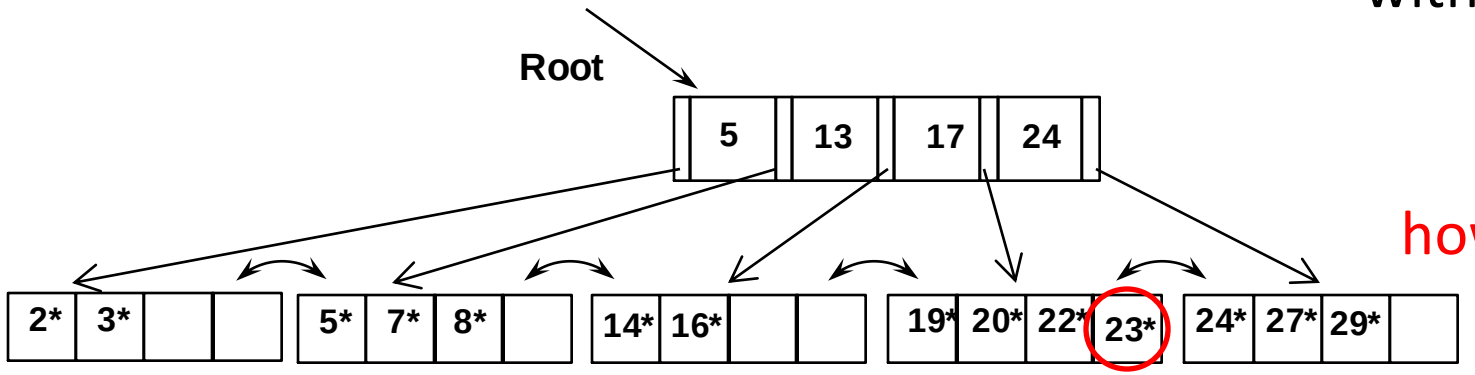
In this example, we can avoid split by re-distributing entries; however, this is usually not done in practice.

how to re-distribute entries?

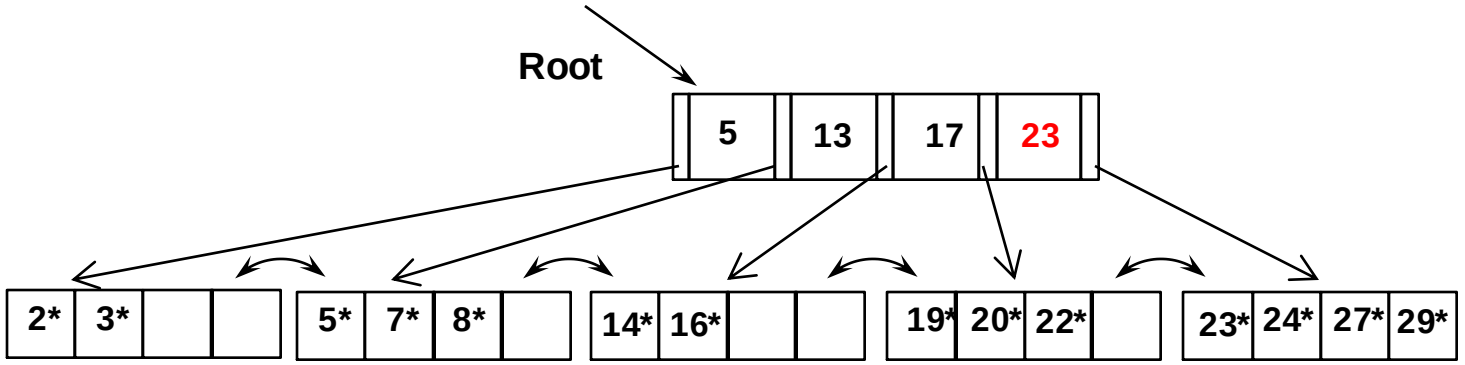
Example B+ Tree - Inserting 21*

with redistribution

how to re-distribute entries?



create space by moving one entry
& update parent node



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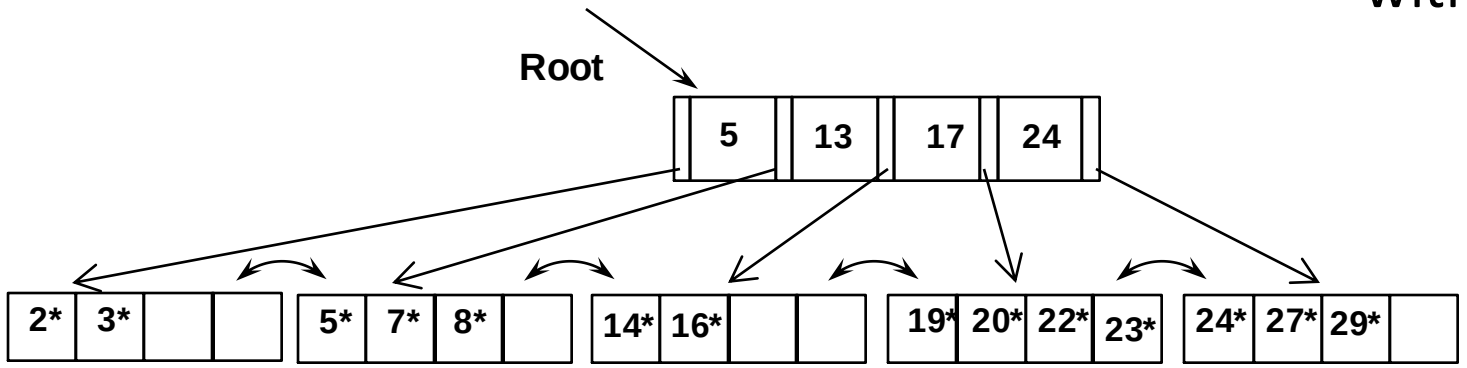
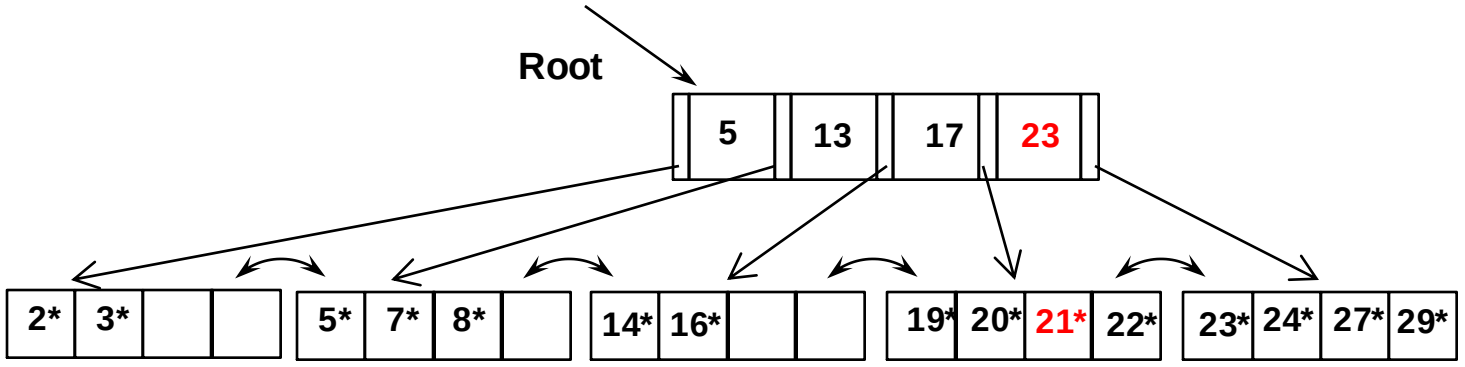
with redistribution

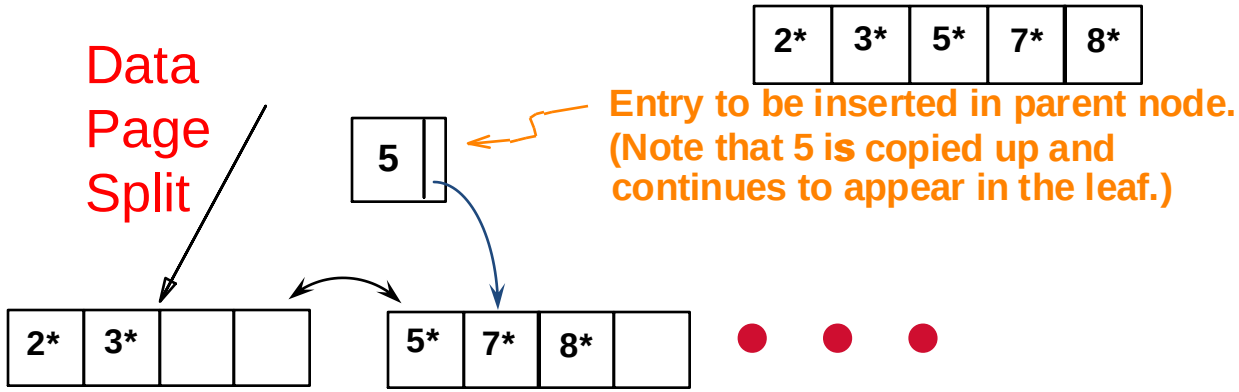




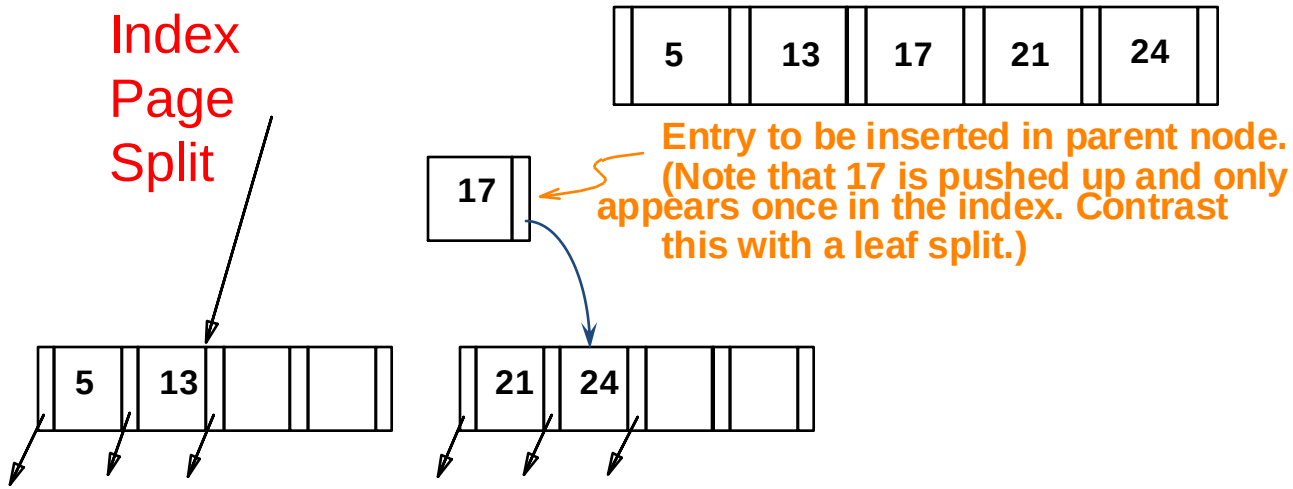
Example: Data vs. Index Page Split

minimum occupancy is guaranteed in both leaf and index page splits

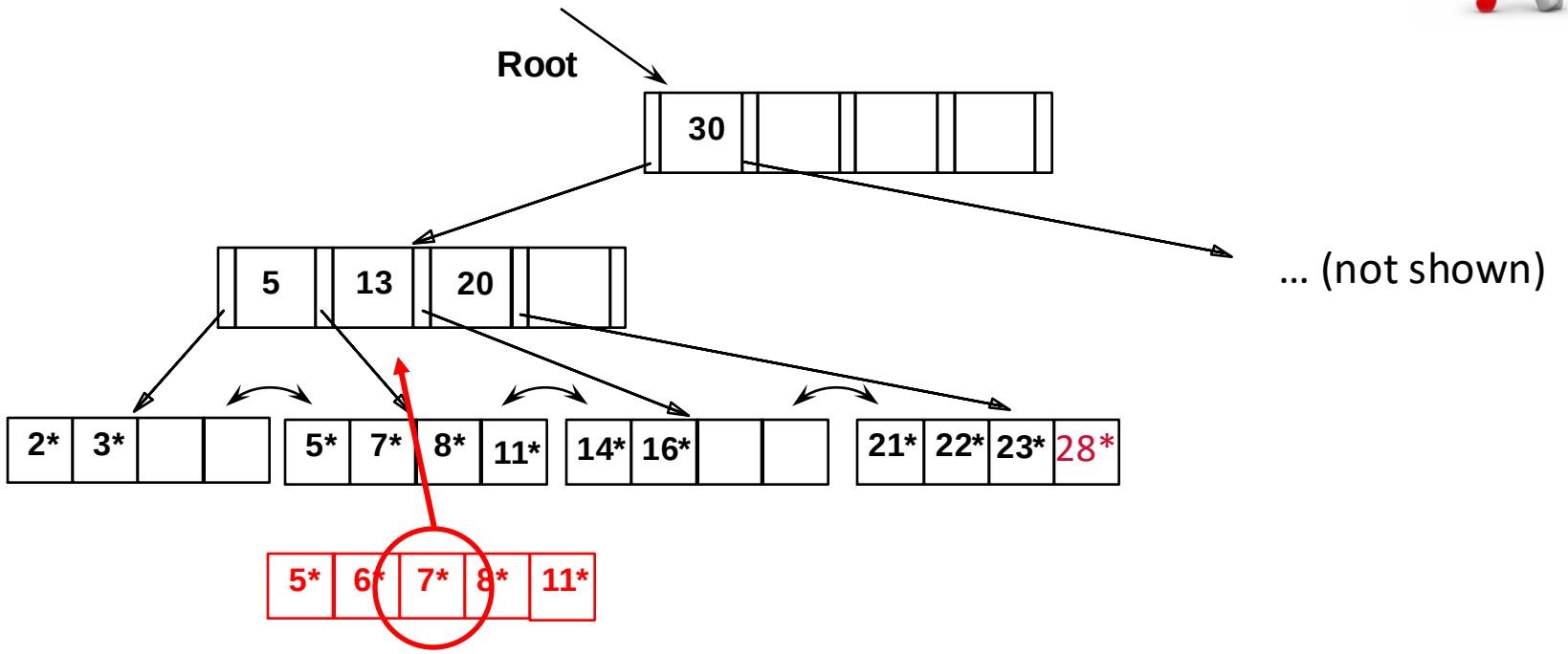
copy-up for data page splits



push-up for index page split



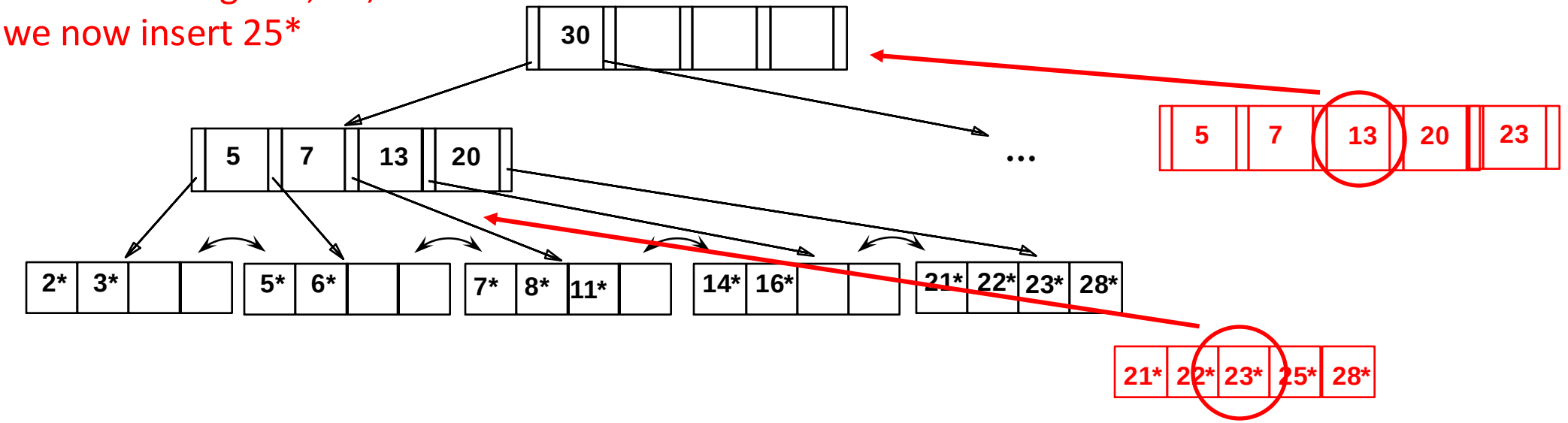
Now you try…



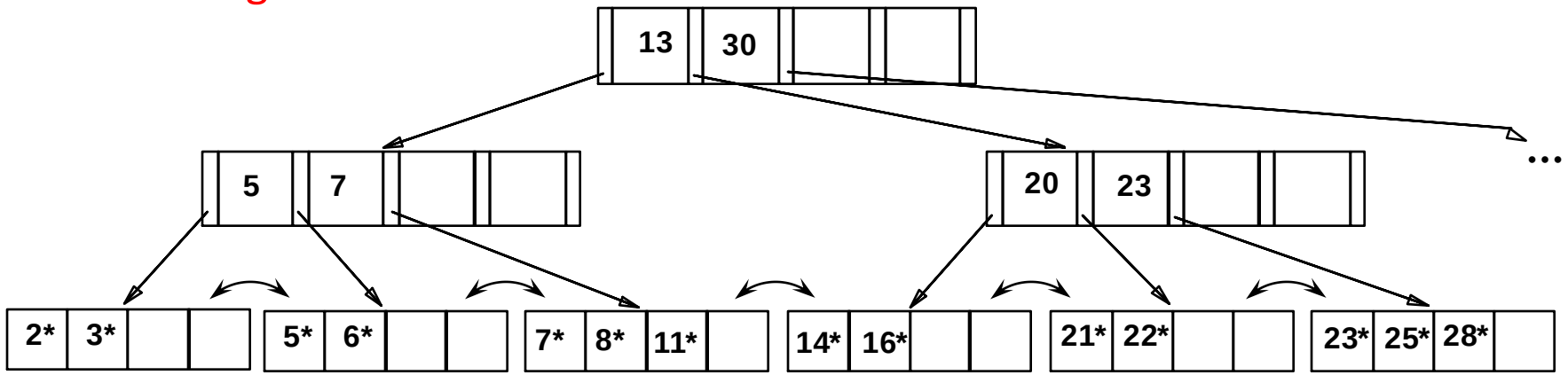
Insert the following data entries (in order): 28*, 6*, 25*

Answer…

After inserting 28*, 6*,
we now insert 25*



After inserting 25*



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Deleting a Data Entry from a B+ Tree

Start at root, find leaf *L* where entry belongs.

Remove the entry.

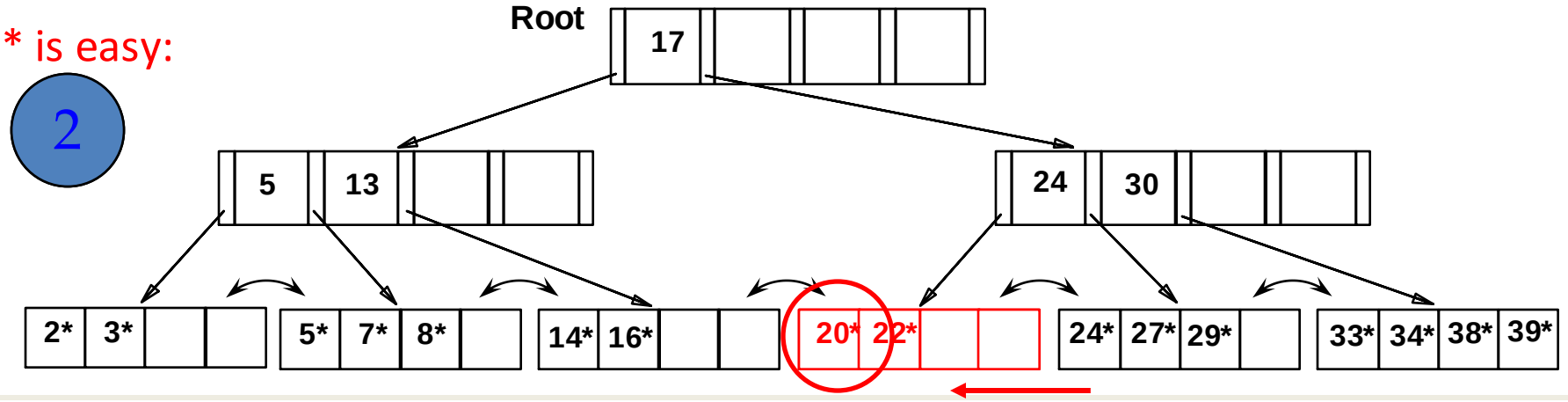
- If L is at least half-full, *done!*
- If L has only **d-1** entries,
  - Try to re-distribute, borrowing from *sibling (adjacent node with same parent as L)*.
  - If re-distribution fails, *merge* *L* and sibling.

If merge occurred, must delete entry (pointing to *L* or sibling) from parent of *L*.

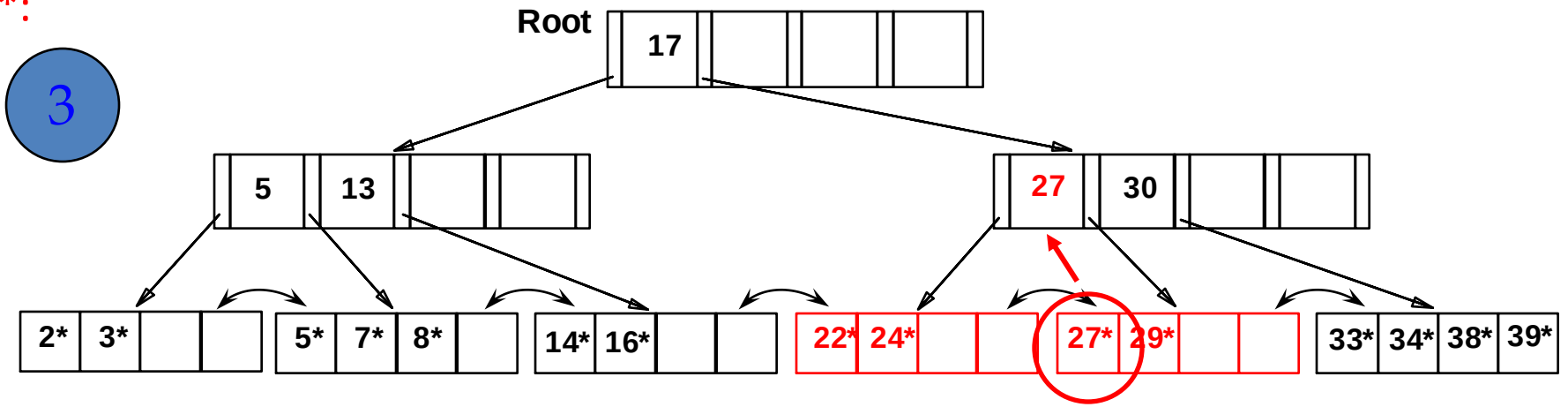
Merge could propagate to root, decreasing height.

Example: Delete 19* & 20*

Deleting 19* is easy:



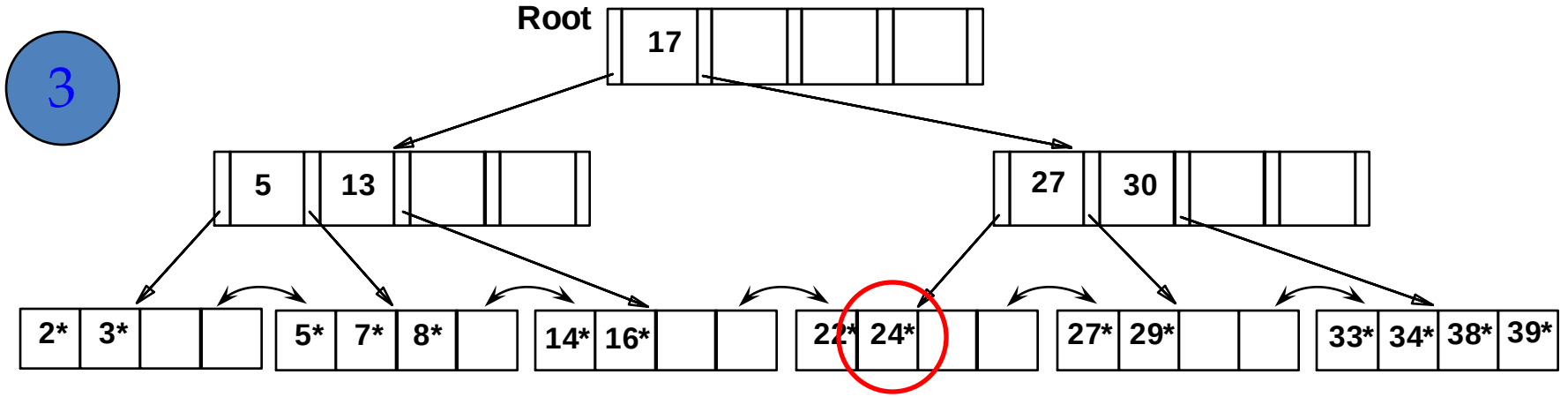
Deleting 20*:



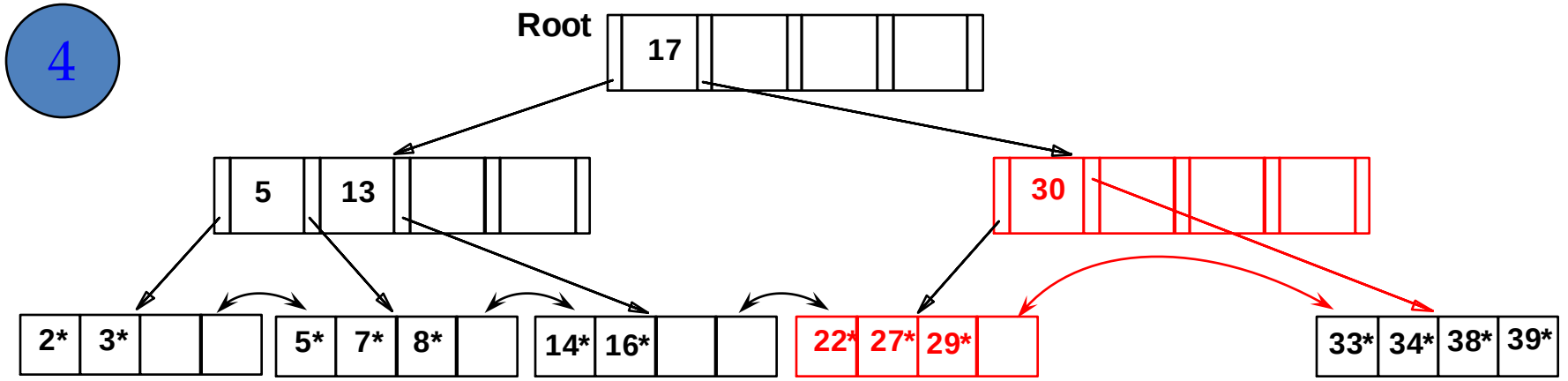
Deleting 20* is done with re-distribution. Notice how middle key is *copied up*.

… and then deleting 24*

3

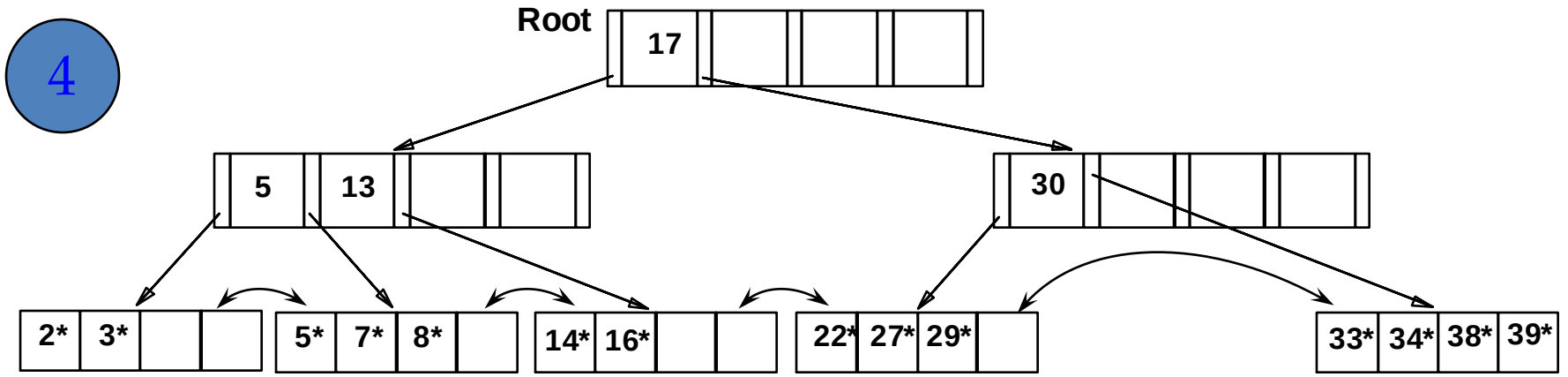


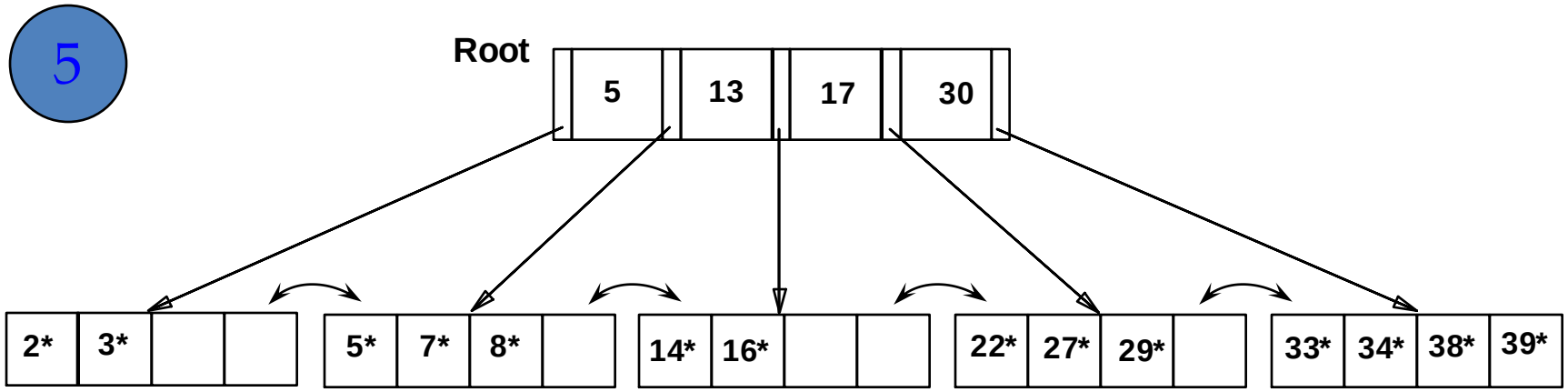
4



Must merge leaves **… but are we done??**

… merge non-leaf nodes, shrink tree



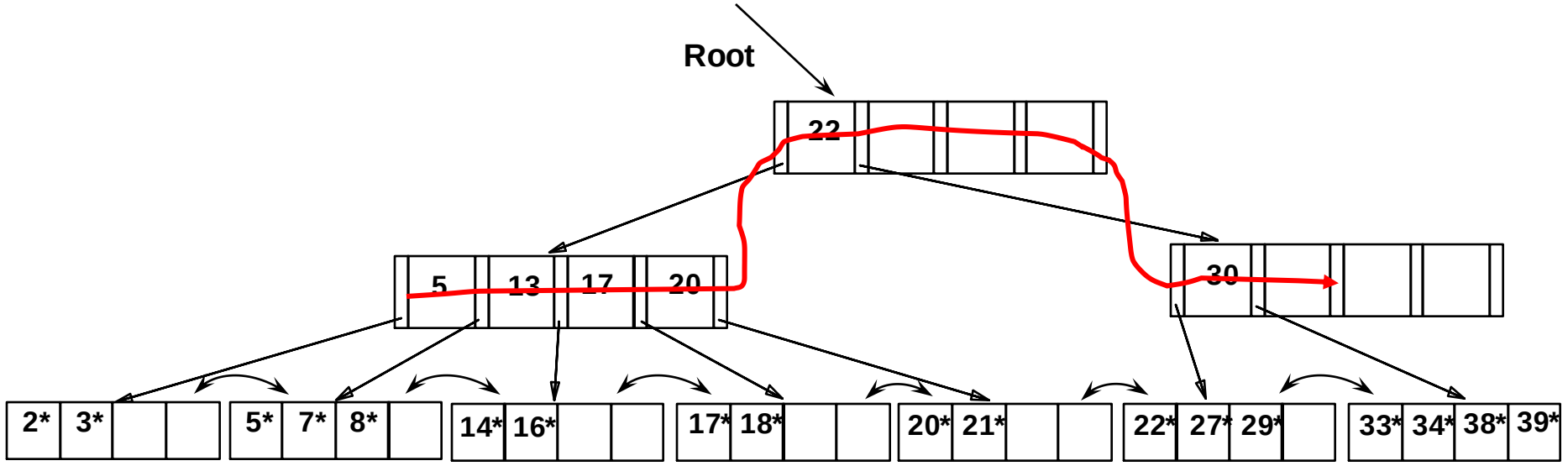


Example of non-leaf re-distribution

Tree is shown below *during deletion* of 24*.

What could be a possible initial tree?

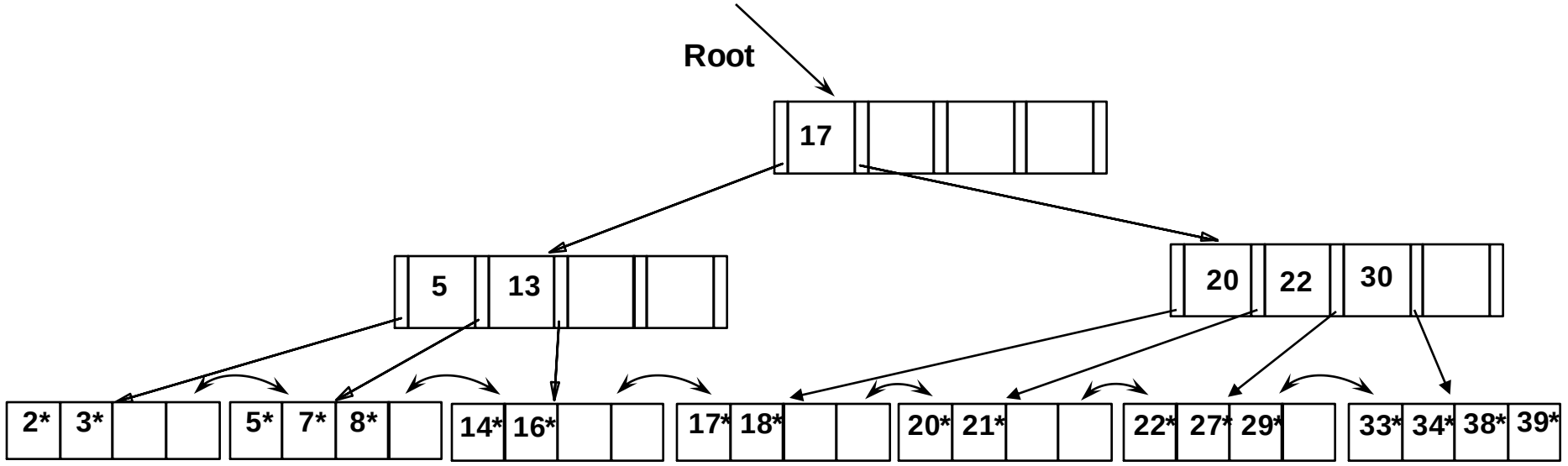
In contrast to previous example, can re-distribute entry from left child of root to right child.



After Re-distribution

Intuitively, entries are re-distributed by "*pushing through*" the splitting entry in the parent node.

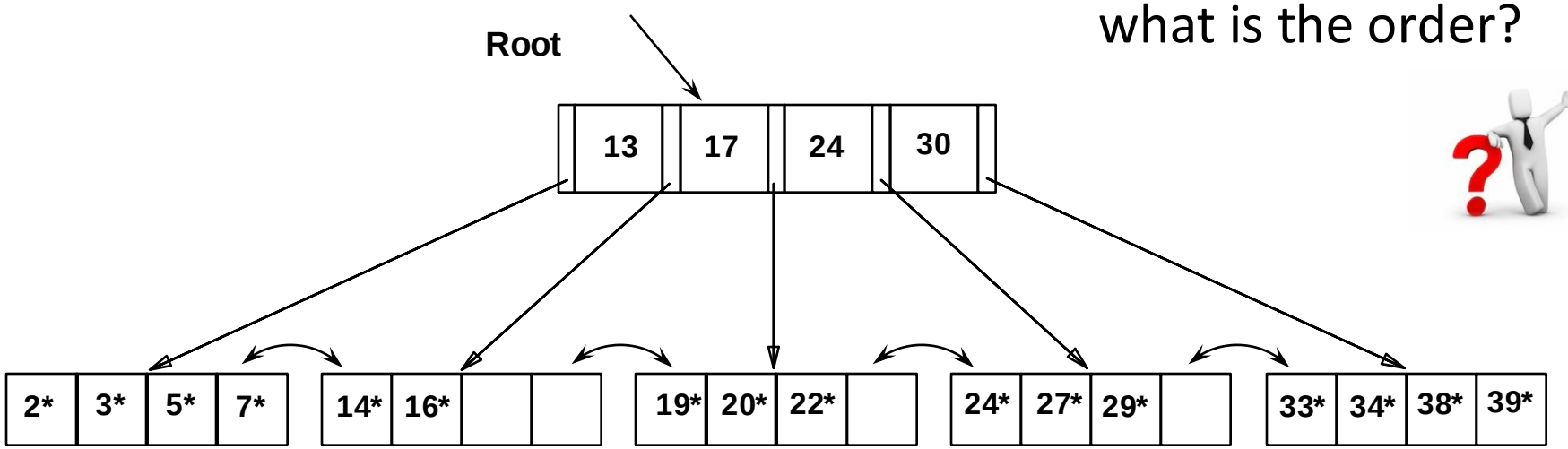
it suffices to re-distribute index entry with key 20;
we have re-distributed 17 as well for illustration



Reminders

begin at root, compare keys to reach the leaf

"order" *d* means d to 2*d elements



what is the order?

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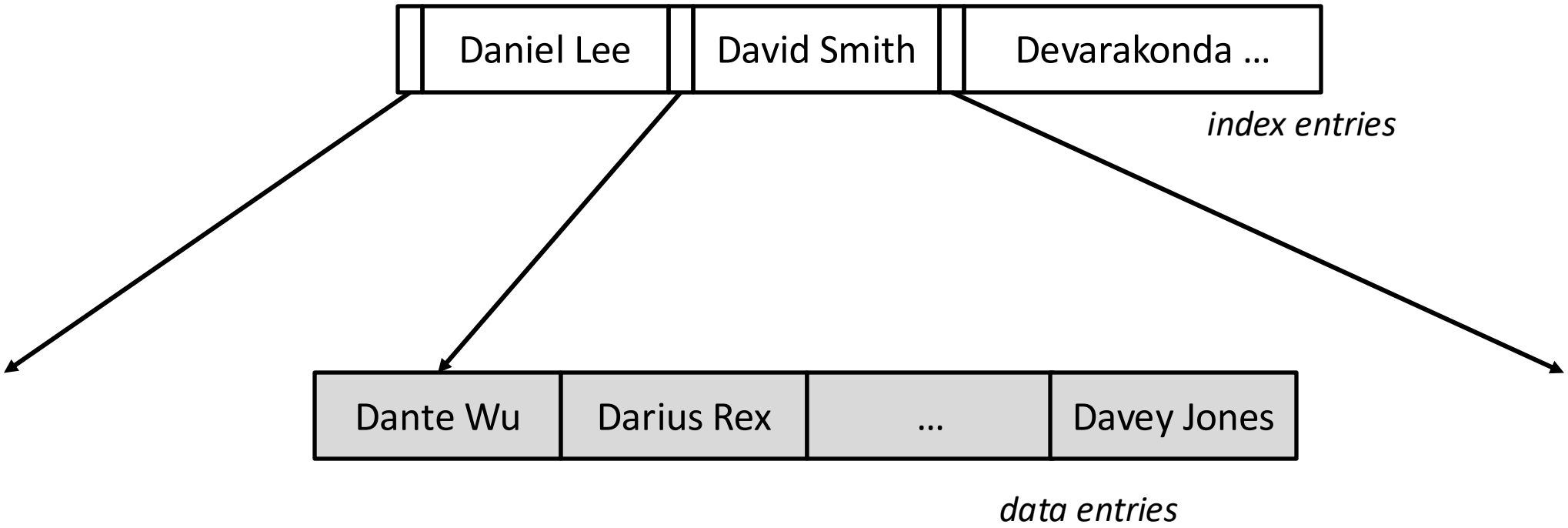
Prefix Key Compression

we want to increase fan-out

why?

key values in index entries (internal nodes) are used to "direct traffic"

how to compress?



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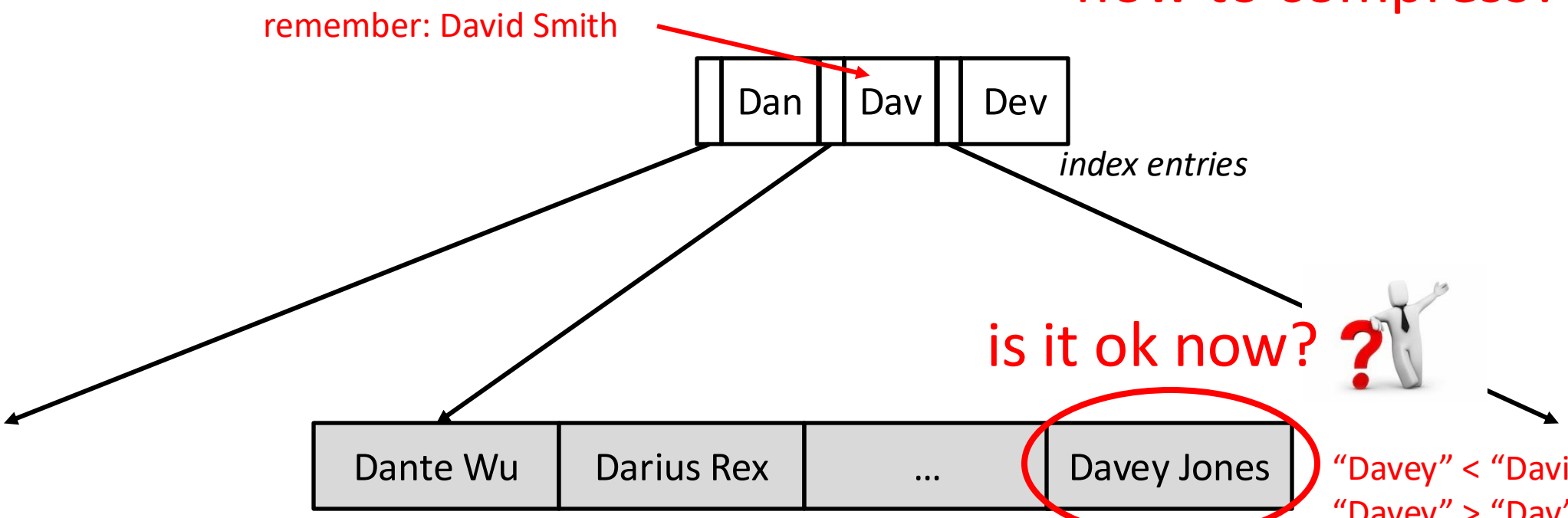

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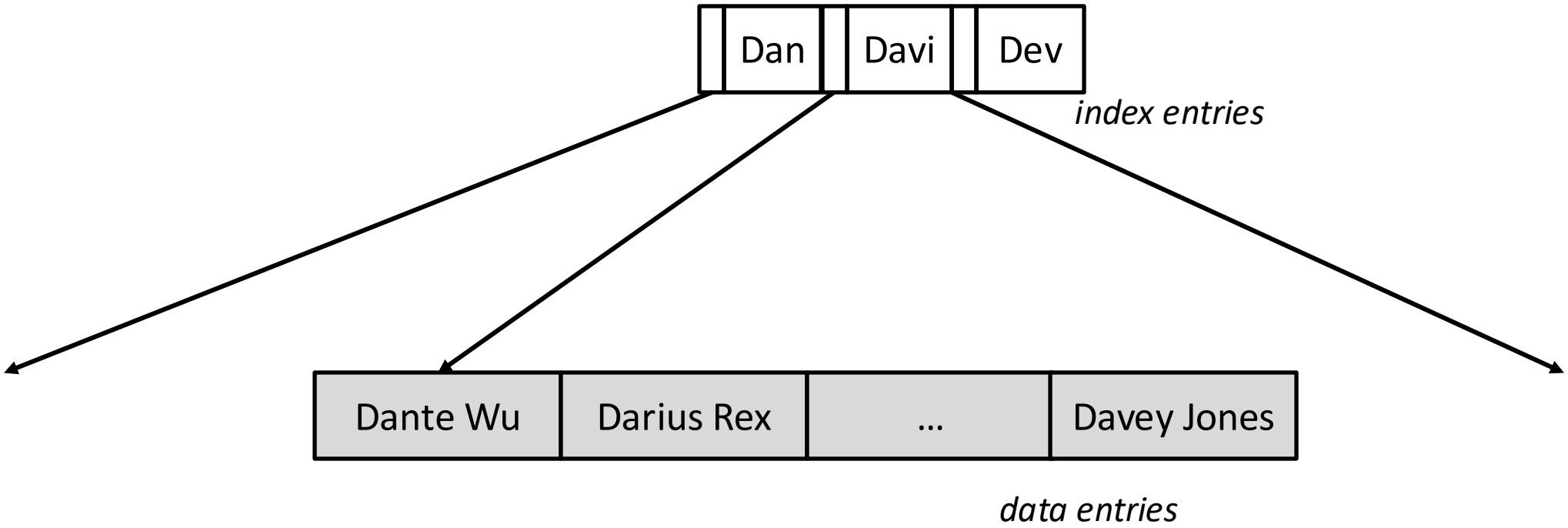


Prefix Key Compression

we want to increase fan-out

keys in index entries (internal nodes) are used to “direct traffic”

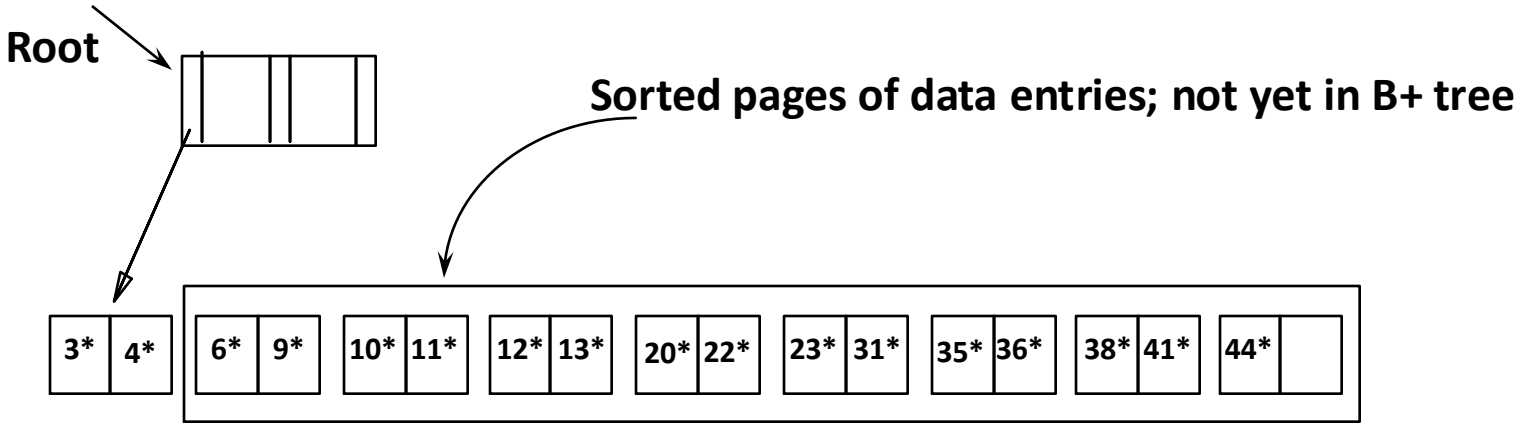
insert/delete must be suitably modified

Bulk Loading of a B+ Tree

If we have a large collection of records, and we want to create a B+ tree on some field, doing so by repeatedly inserting records is very slow.

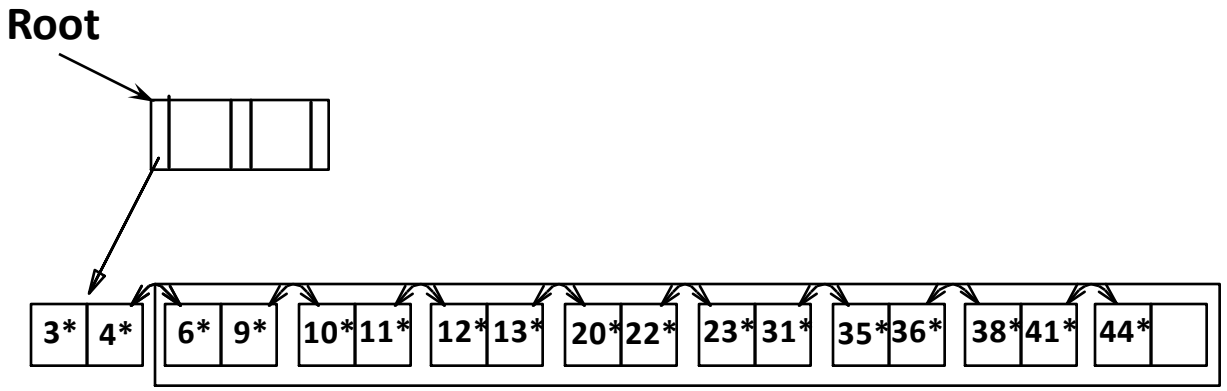
Bulk Loading can be done much more efficiently.

Initialization: Sort all data entries, insert pointer to first (leaf) page in a new (root) page.



Bulk Loading (Contd.)

where to insert: into right-most index page just above leaf level
what to insert: the left-most value of the new leaf



what to do when full? when this fills up, splits node
(if needed split may go up right-most path to the root)

Bulk Loading (Contd.)

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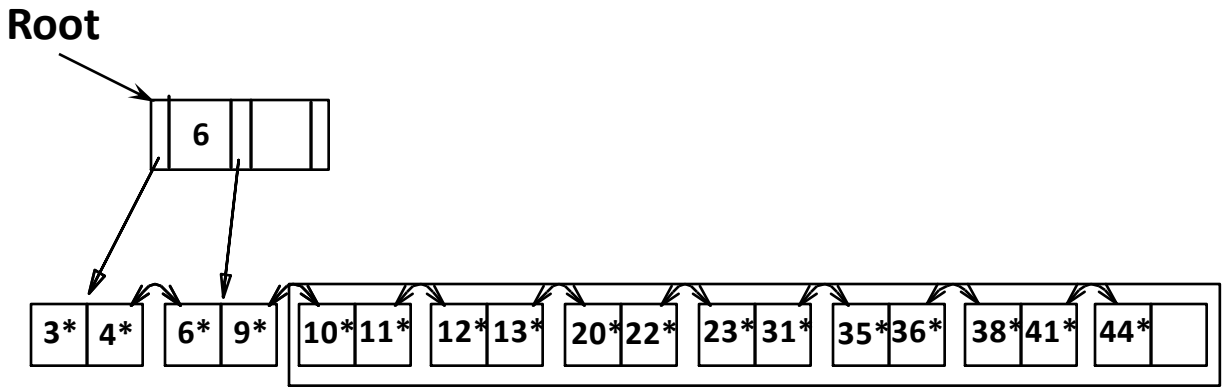

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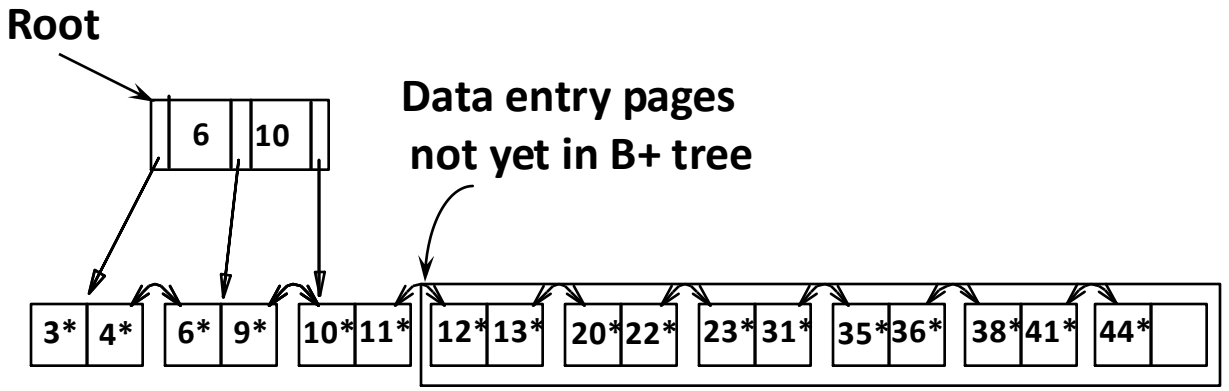

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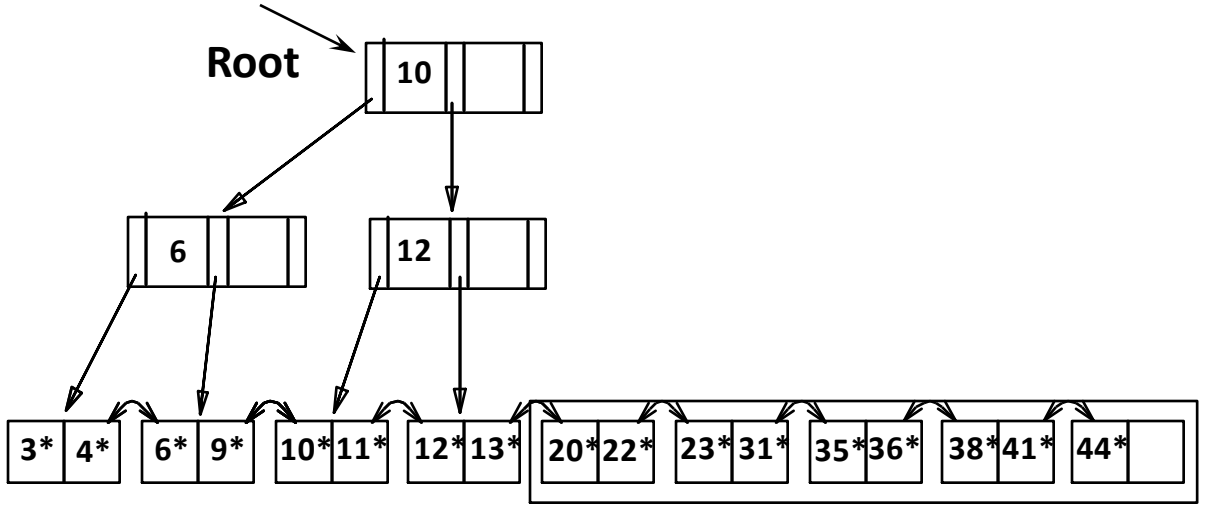

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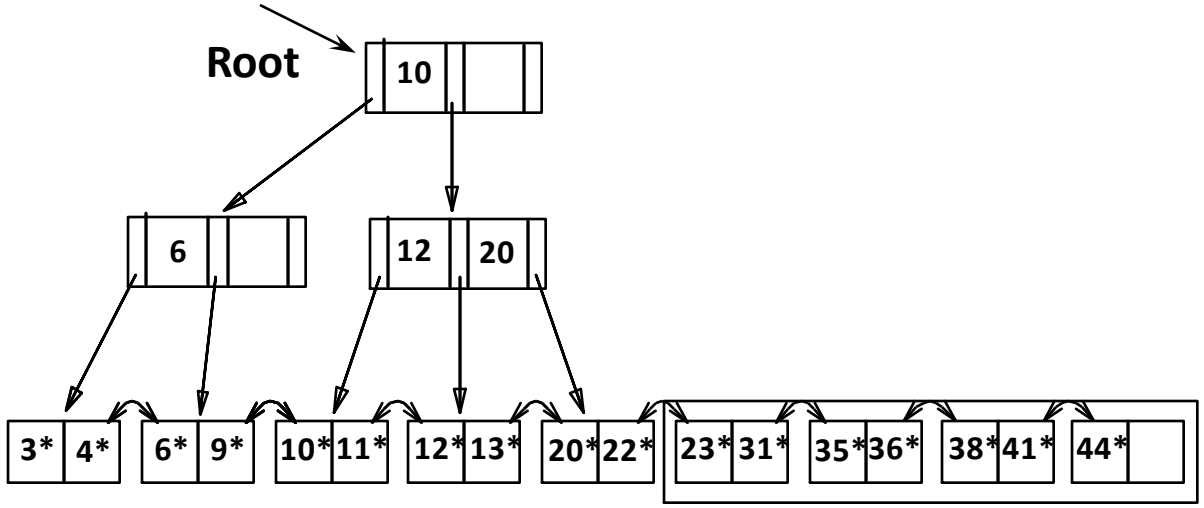

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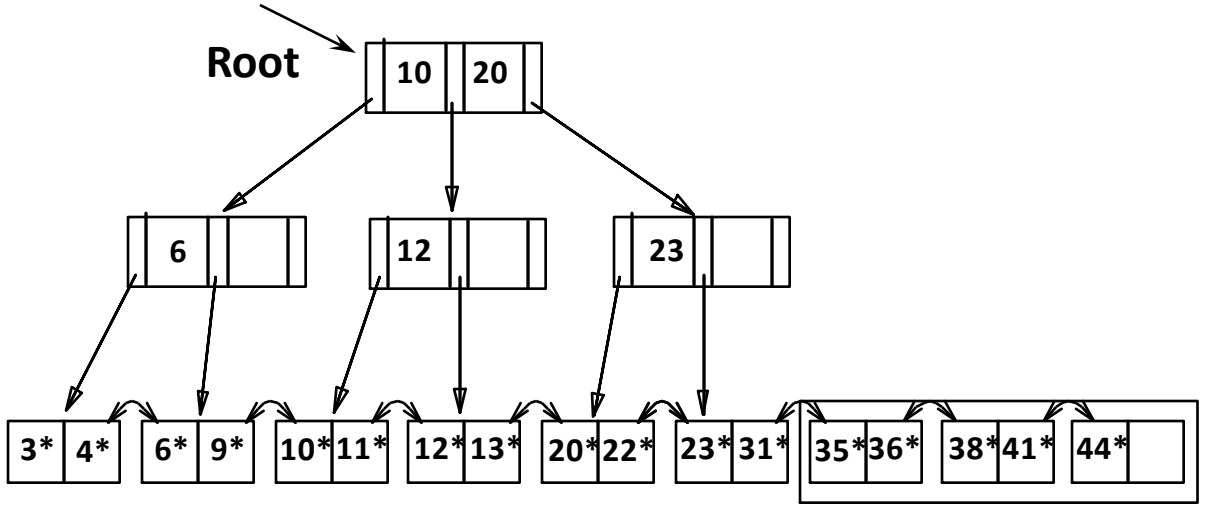

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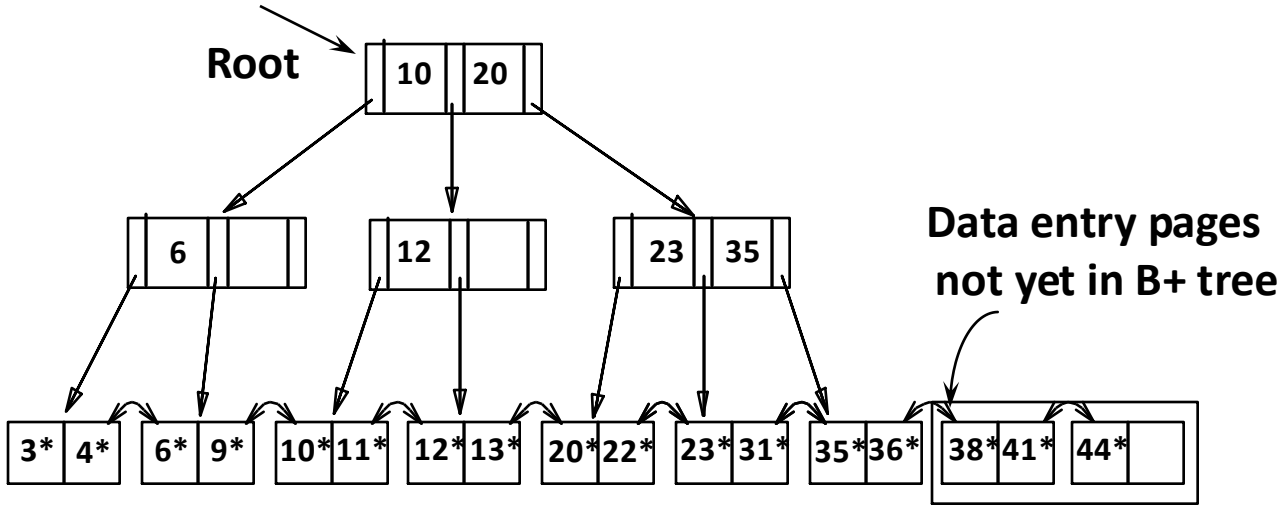

what to do when full? when this fills up, splits node
(if needed split may go up right-most path to the root)

Bulk Loading (Contd.)

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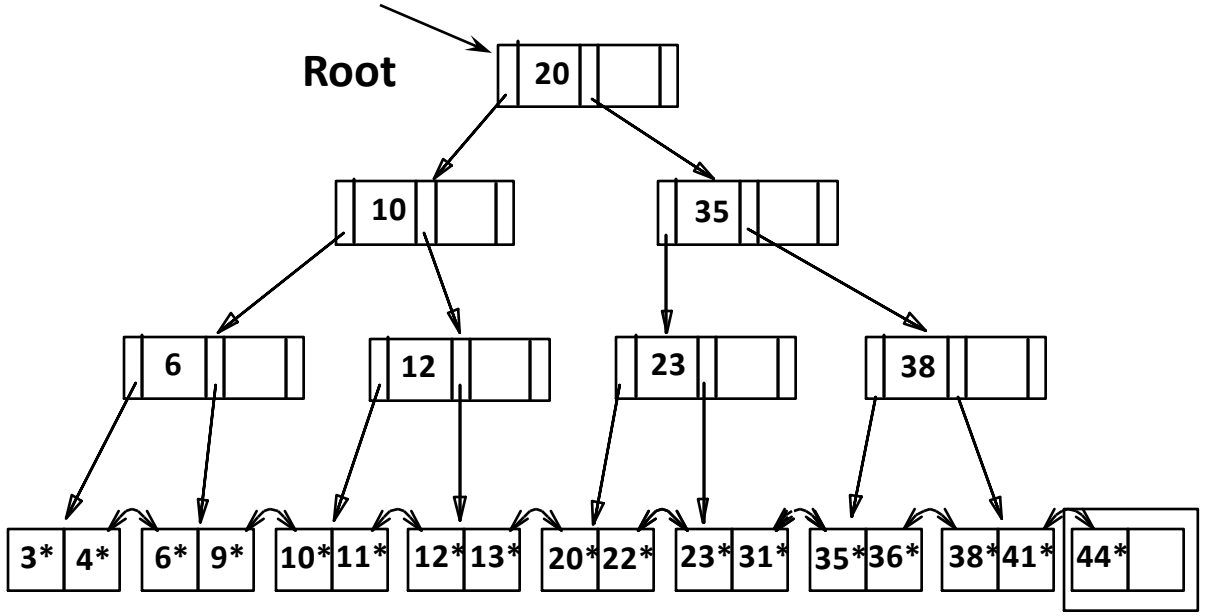

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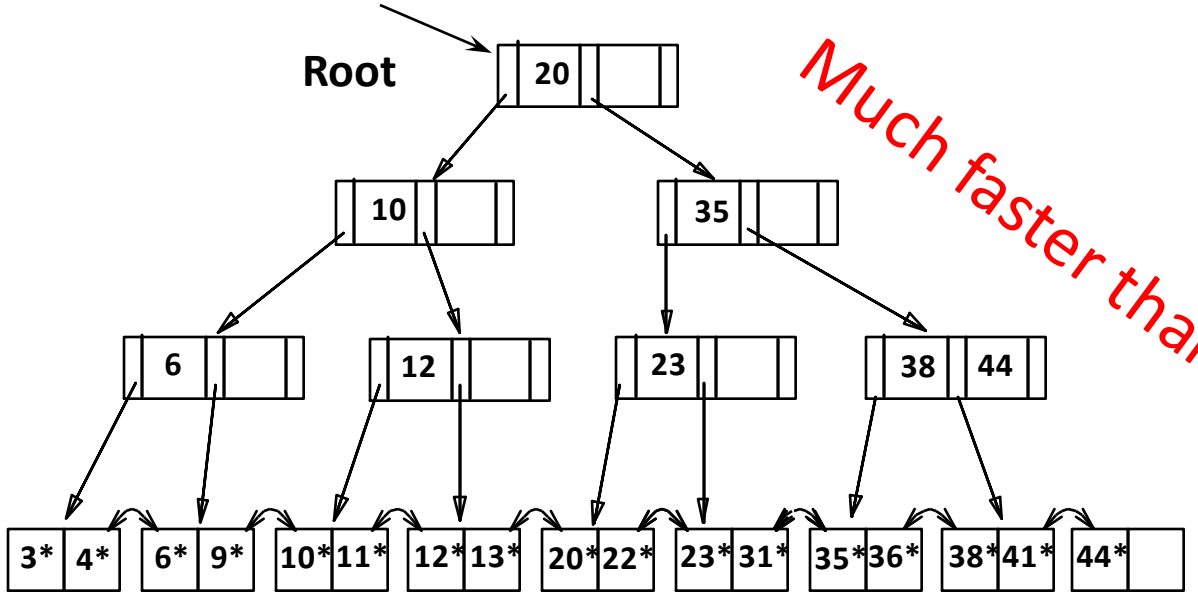

Much faster than inserts!!!

what to do when full? when this fills up, splits node
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Bulk Loading (Change Split Ratio)

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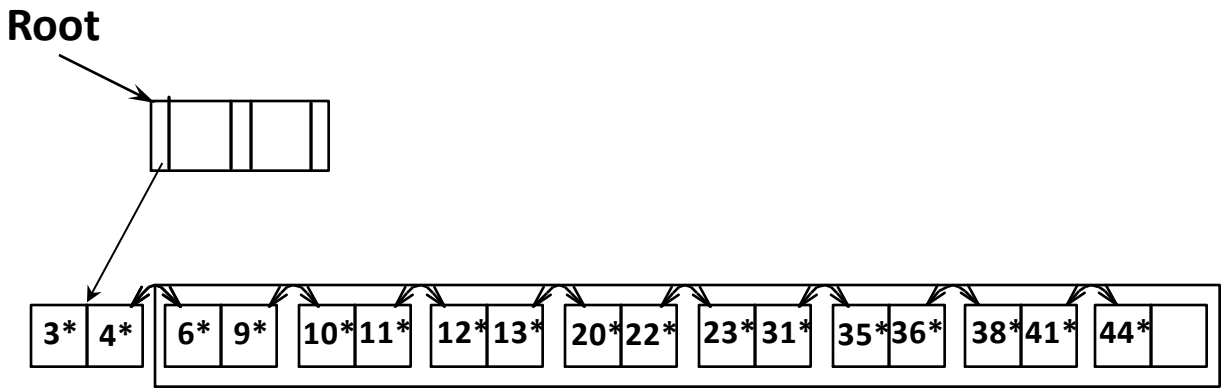

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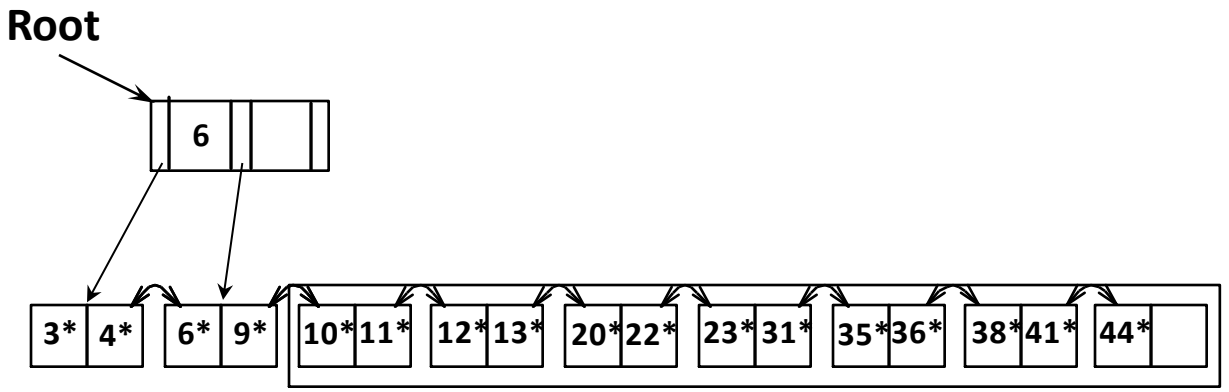

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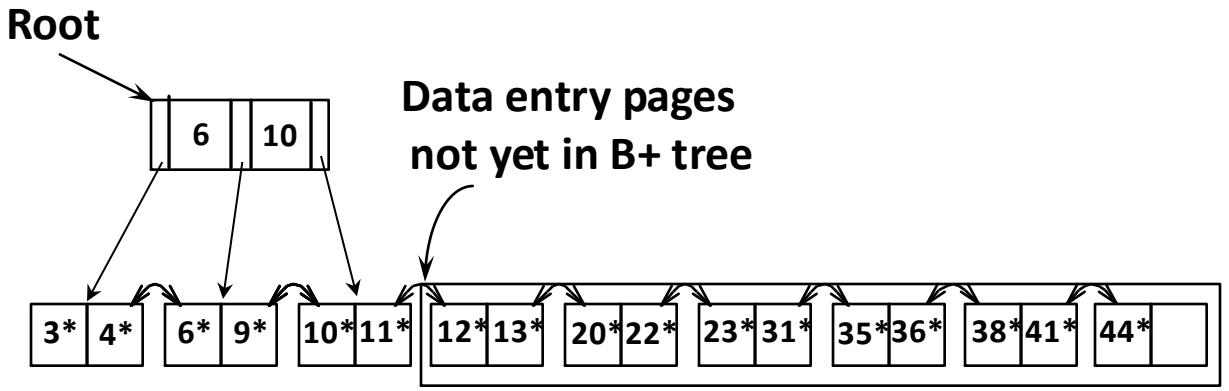

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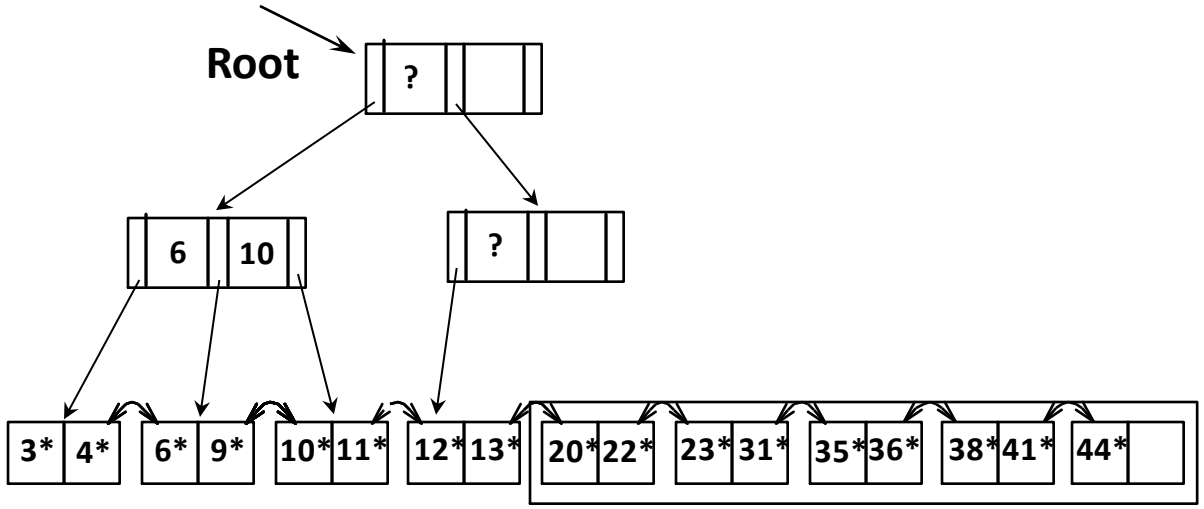

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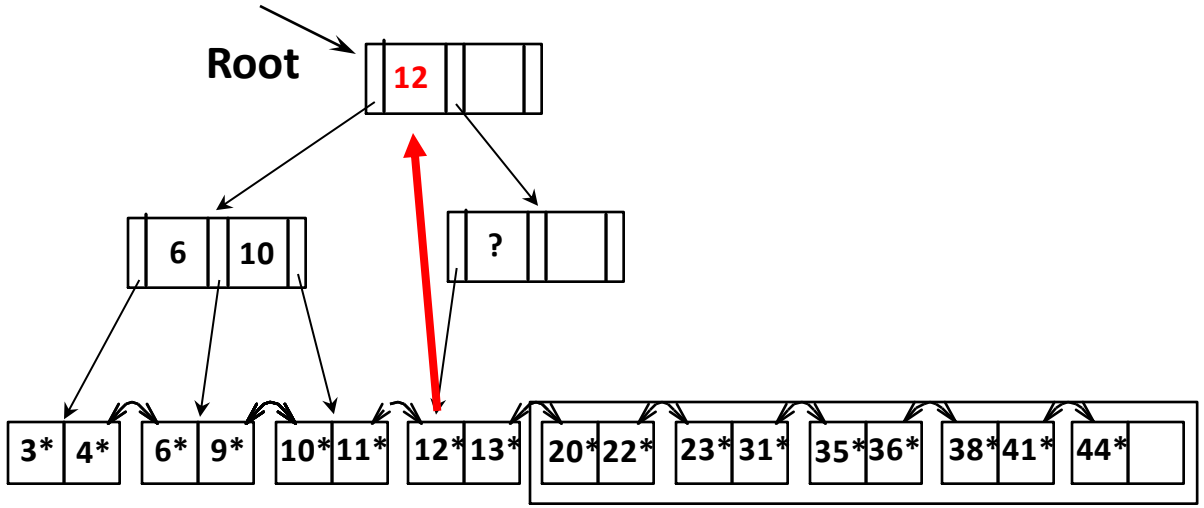

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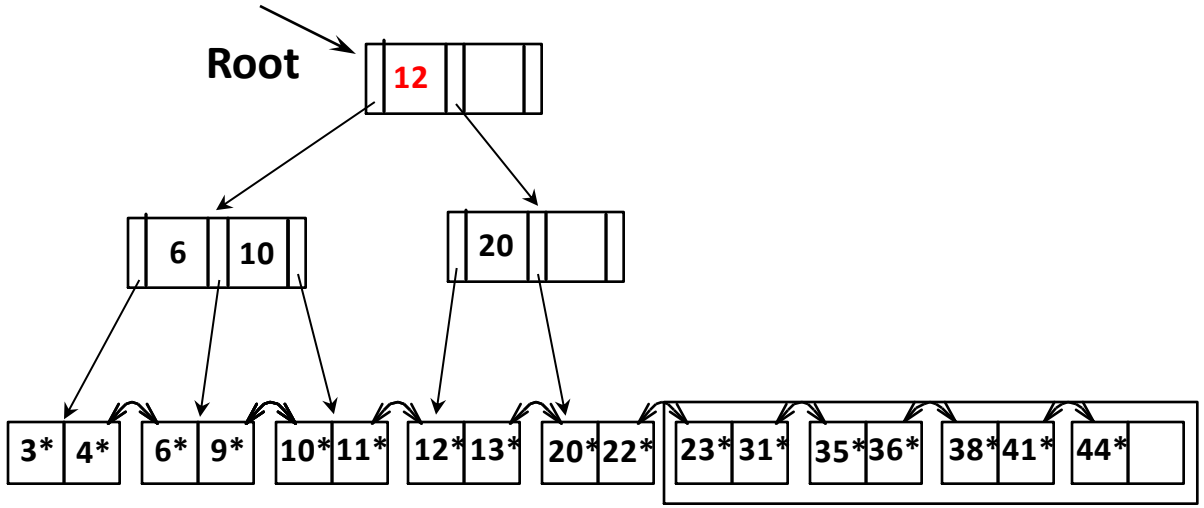

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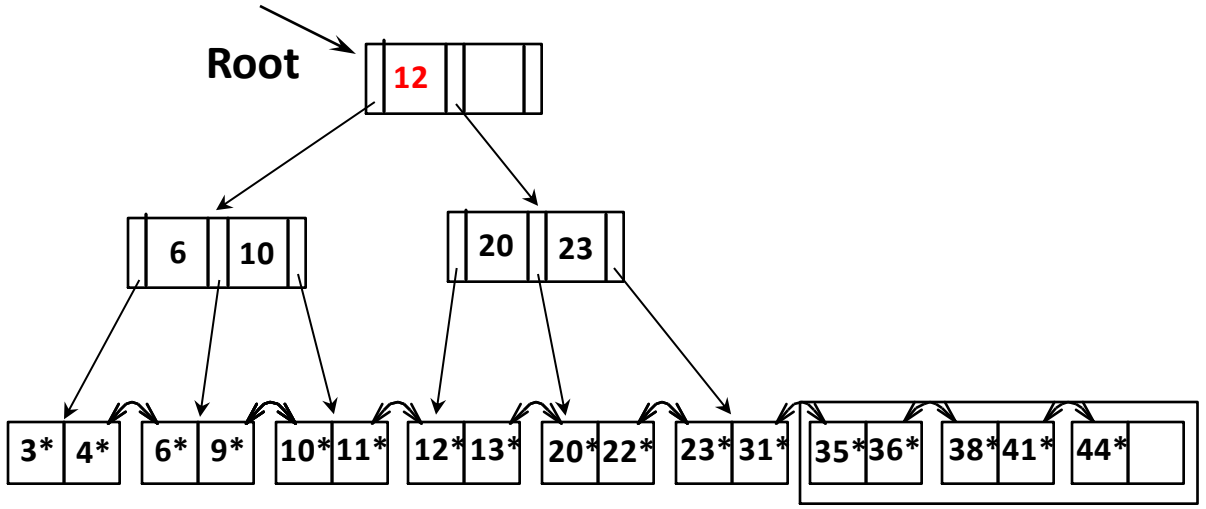

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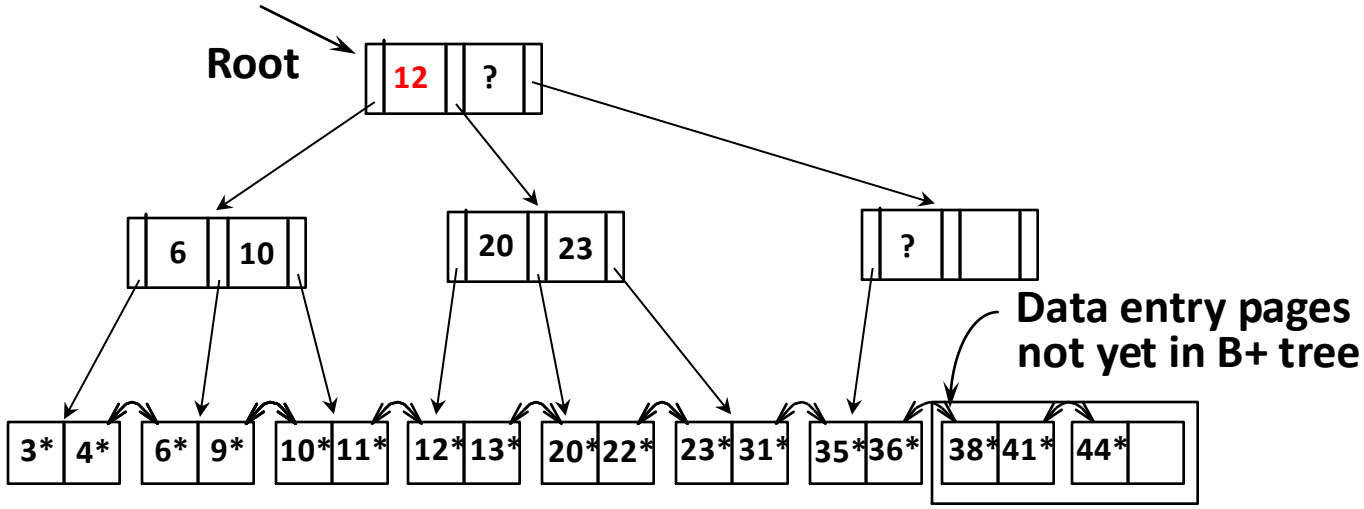

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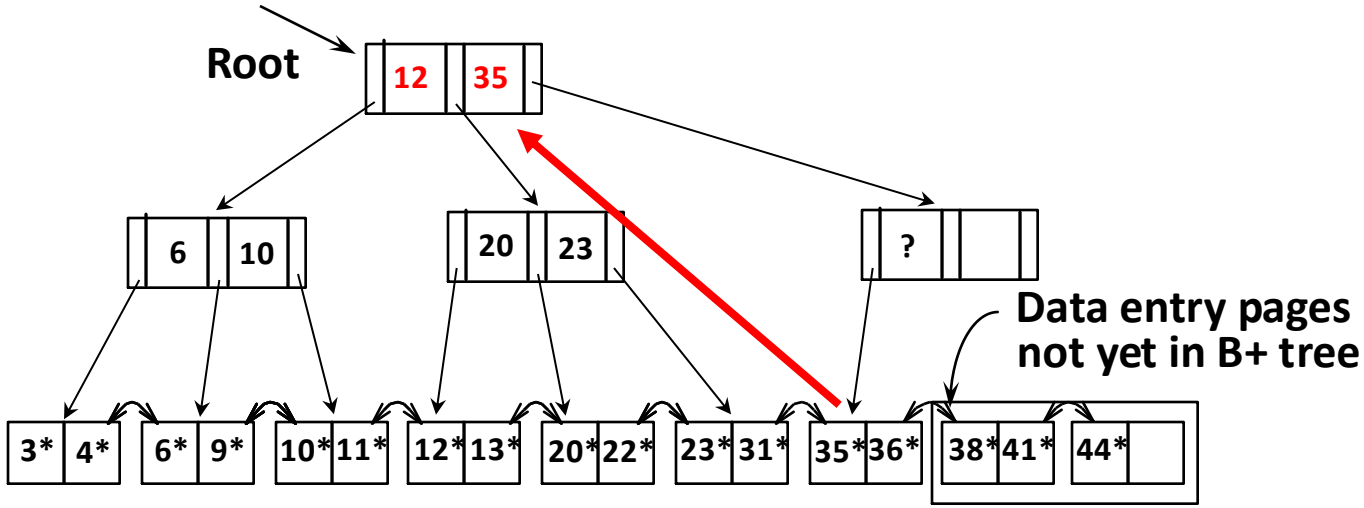

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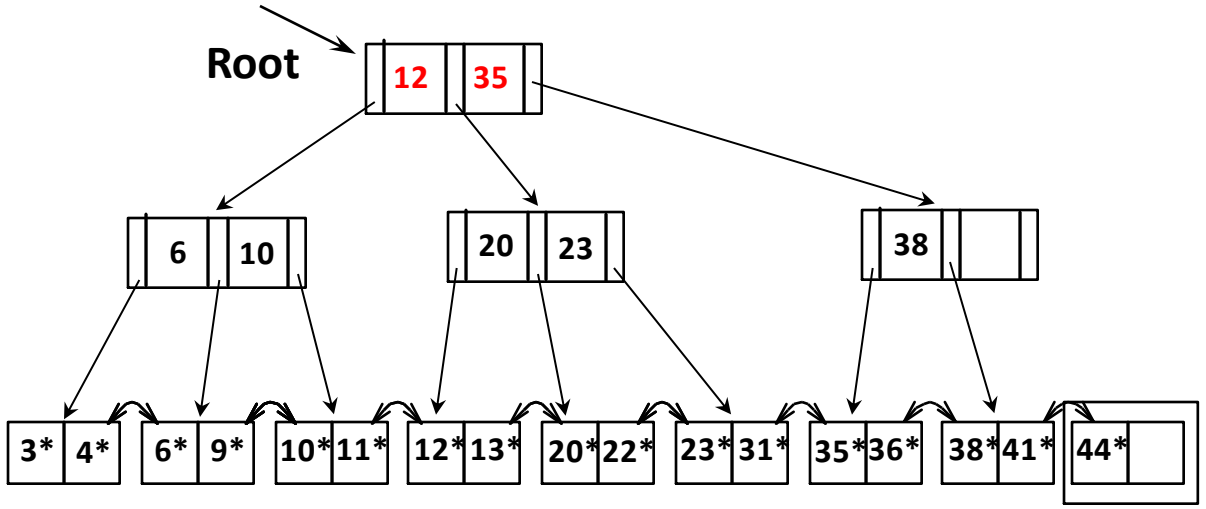

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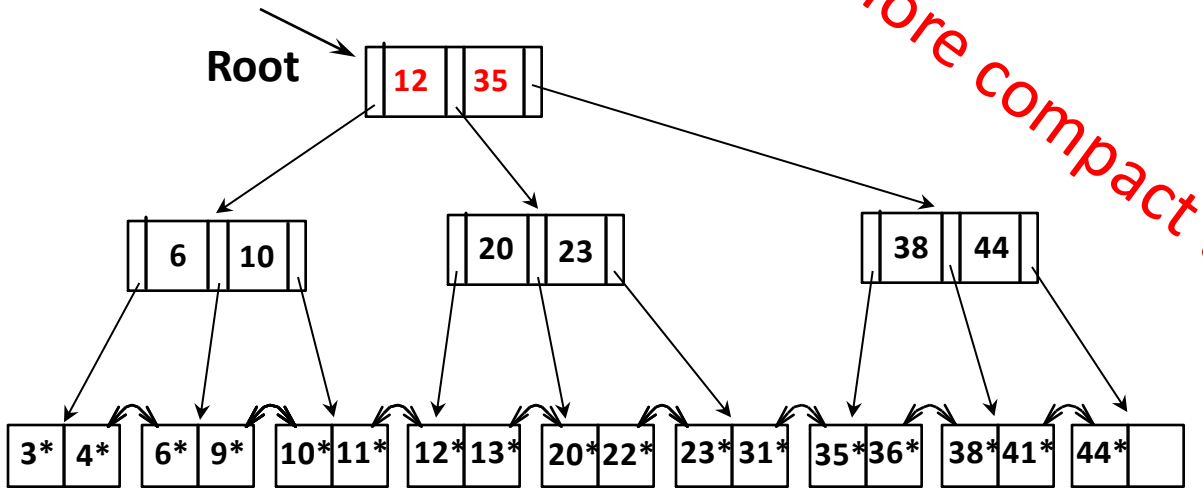


More compact than normal split!

what to do when full? when this fills up, create new empty node and propagate a pointer with the leftmost value of the subtree (if needed this may go up right-most path to the root)

Class Discussion

How can we optimize our tree design if:

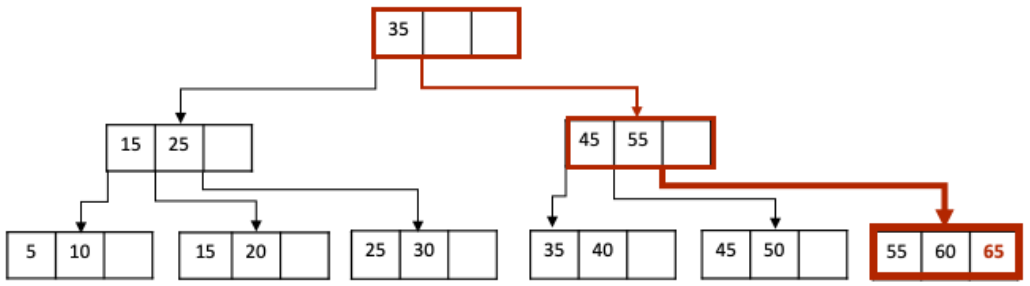
- we are inserting in-order values but we don't always know it
  - Tail Leaf Pointer

- we are inserting some in-order and some out-of-order
  - Last Inserted Leaf Pointer

Inserting into the Tail-leaf (PostgreSQL & MySQL)

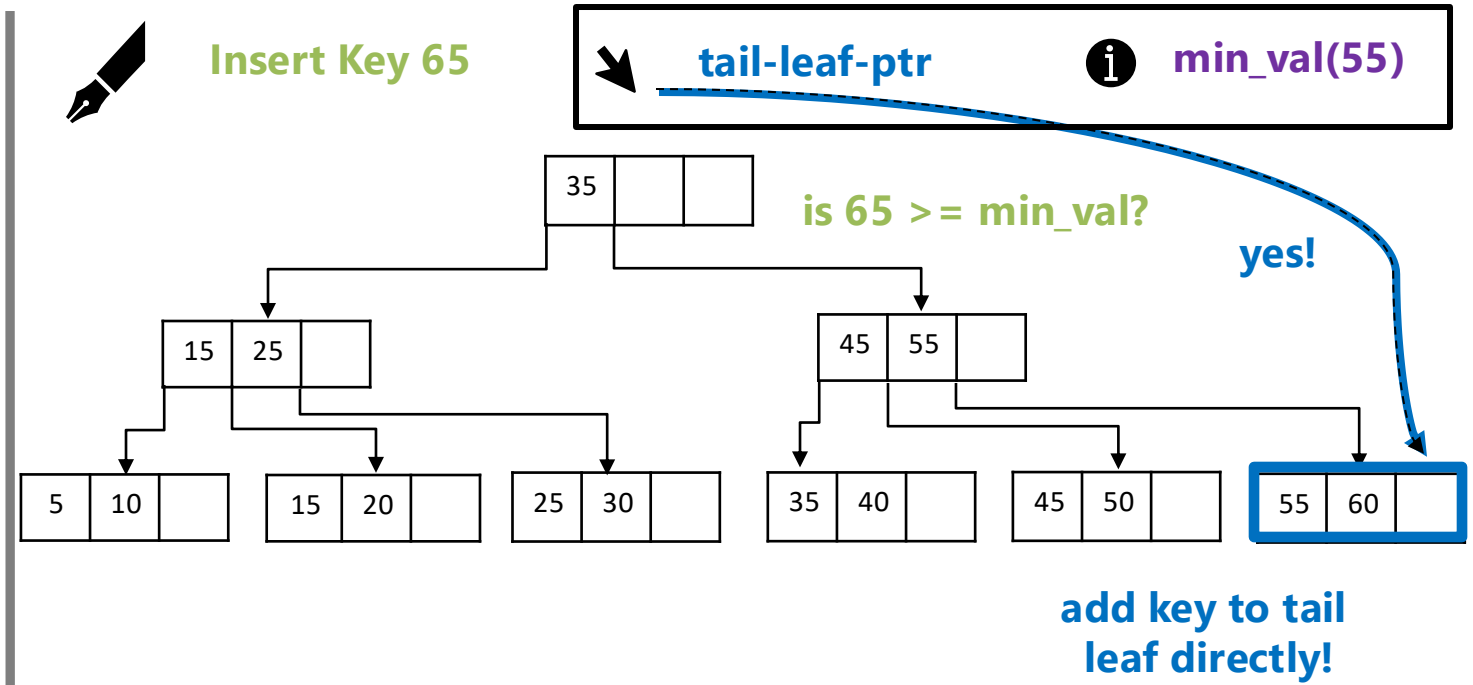
Normal Insertion (top-insert)

Insert Key 65



Tail-leaf Insertion

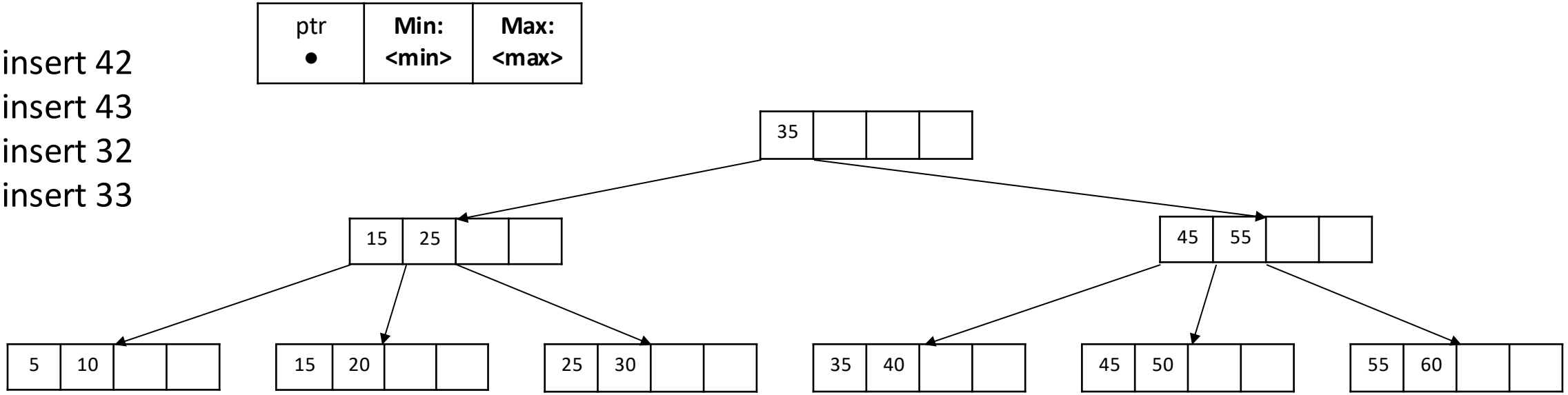
Insert Key 65



add key to tail leaf directly!

Inserting into the Last Inserted Leaf

insert 42
insert 43
insert 32
insert 33



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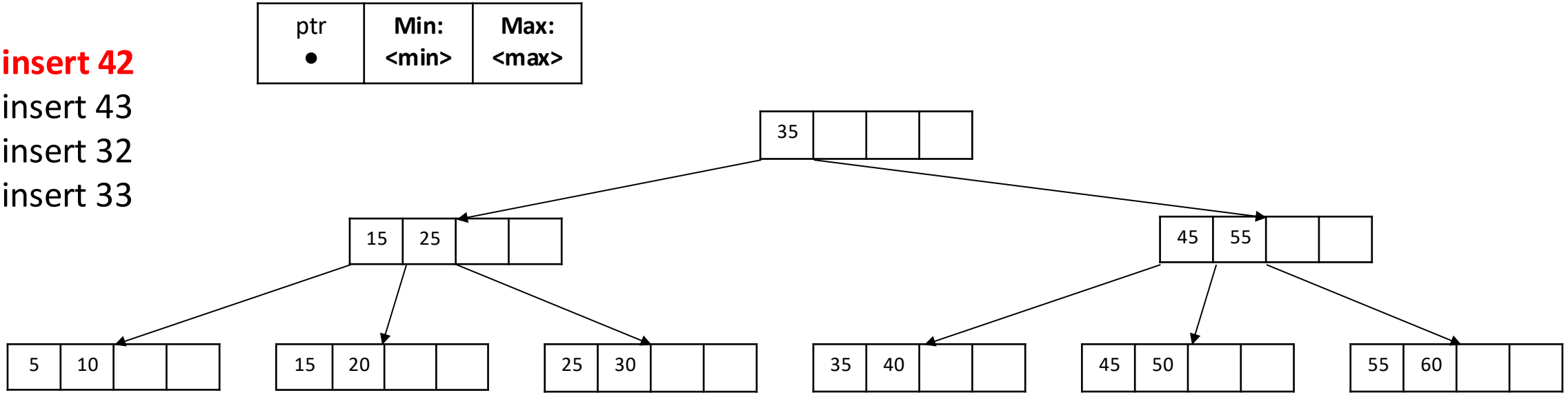

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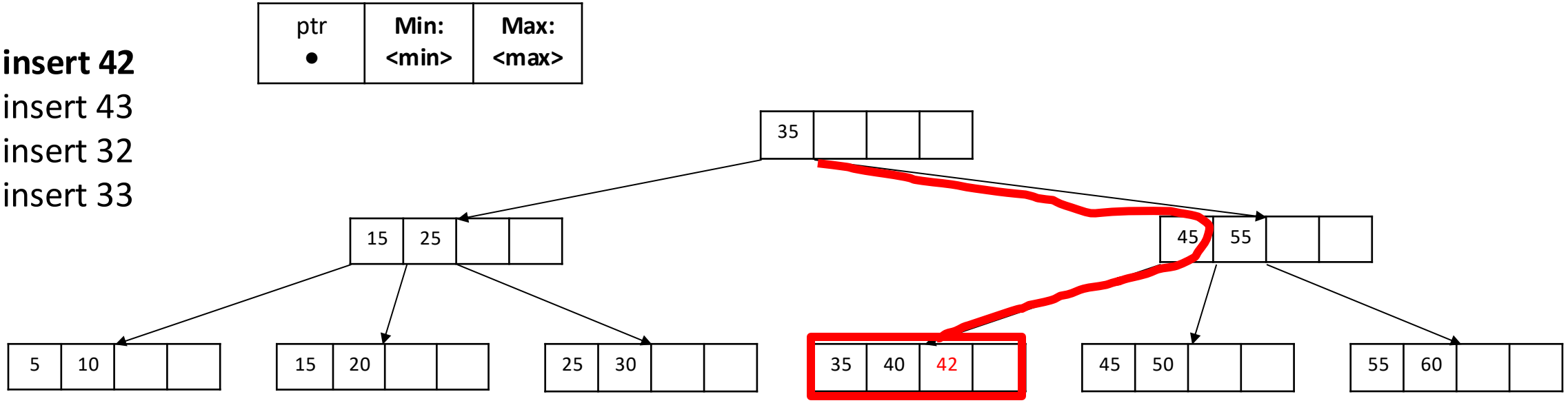

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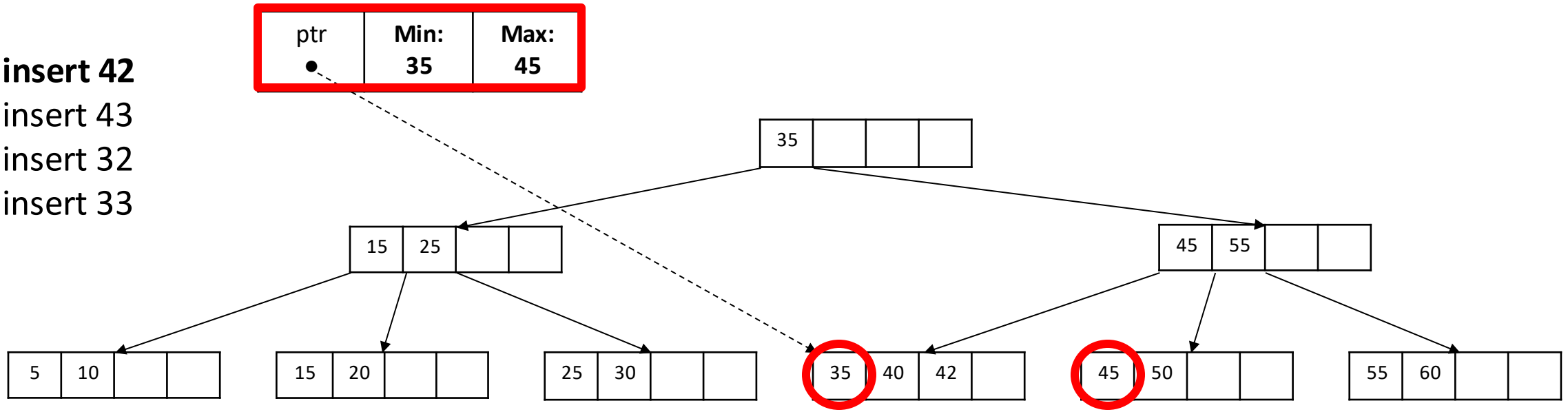

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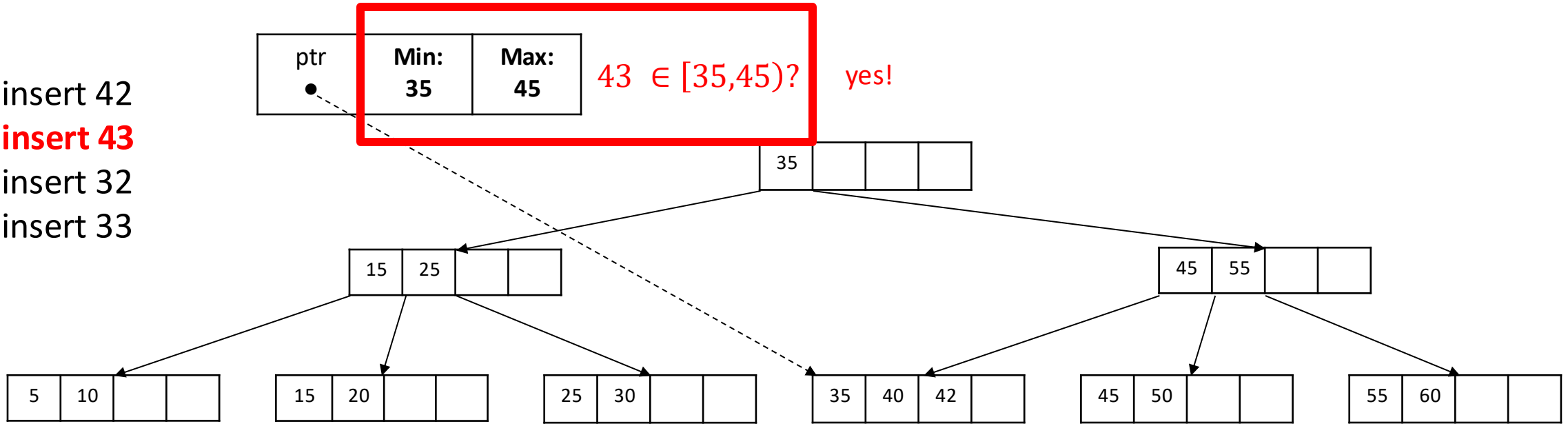

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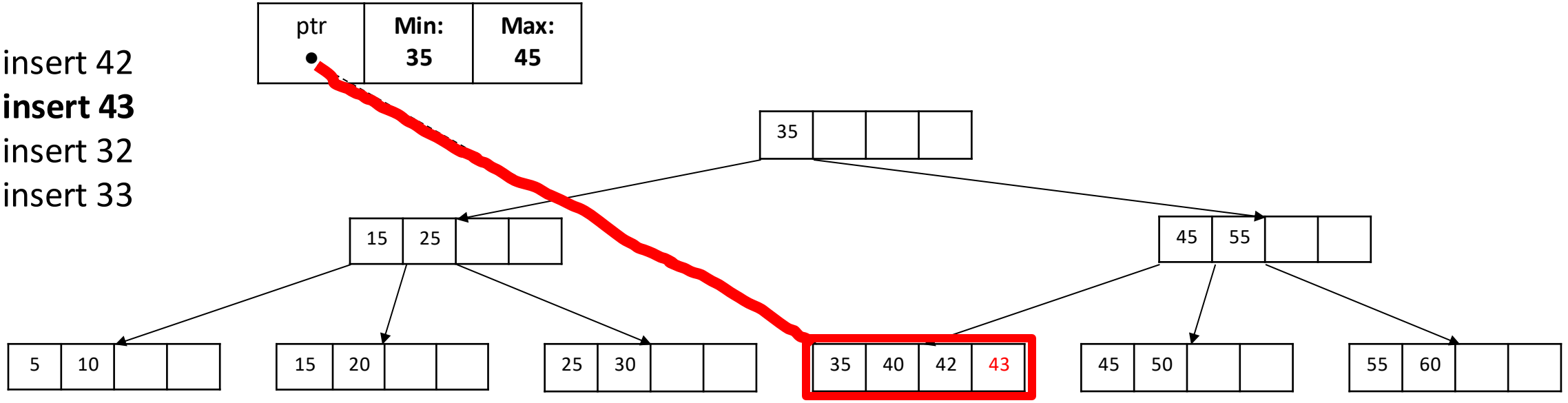

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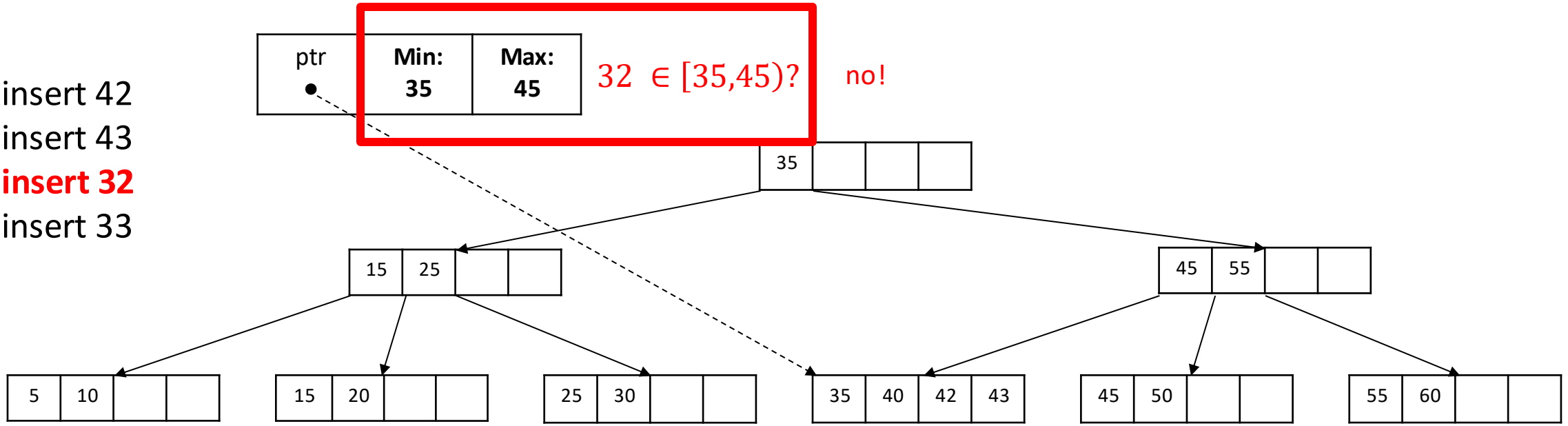

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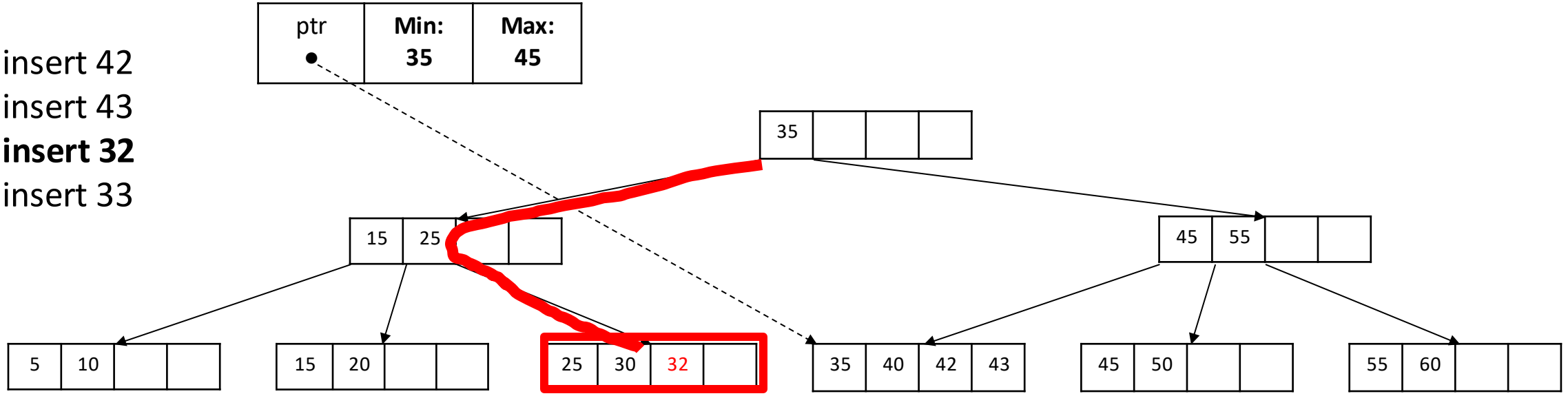

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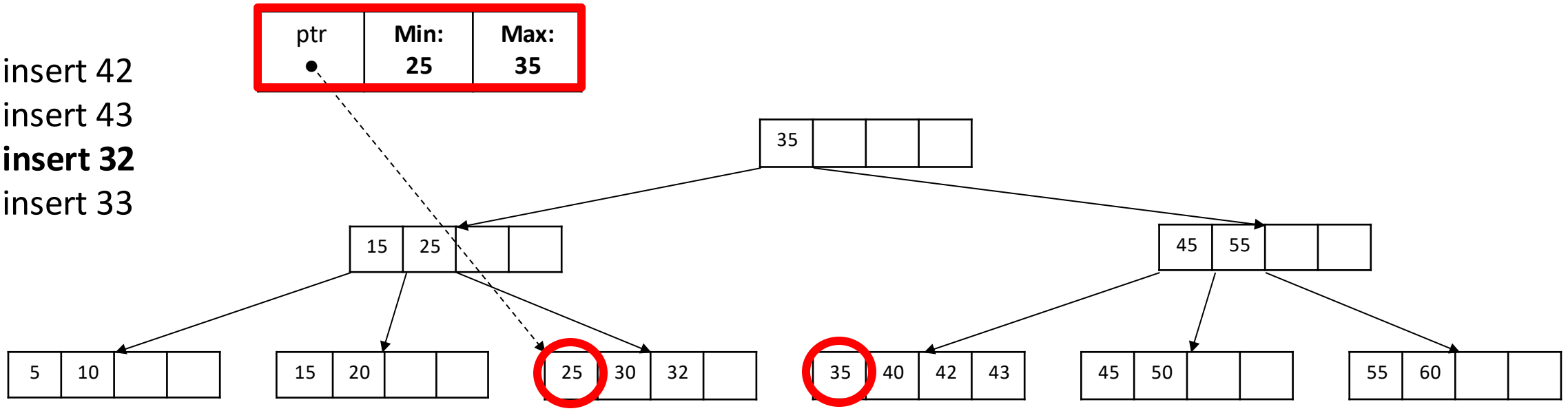

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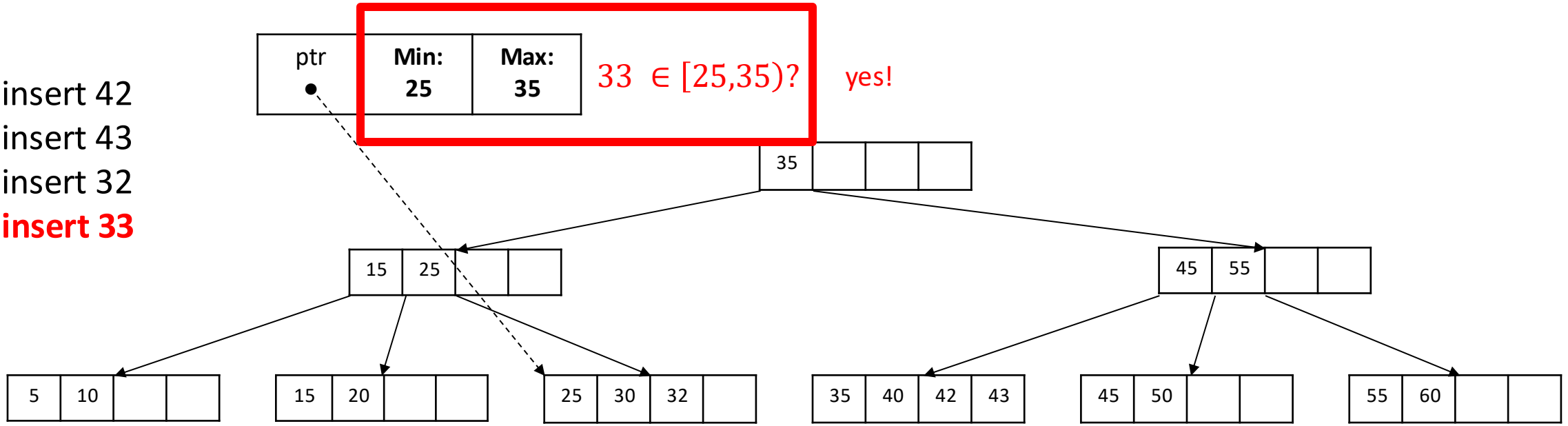

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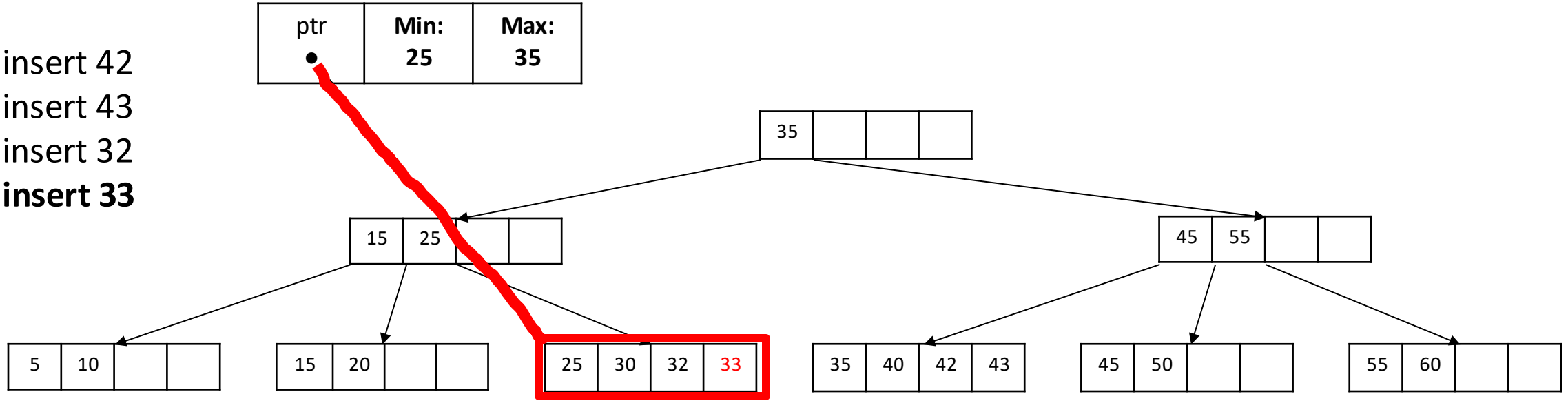
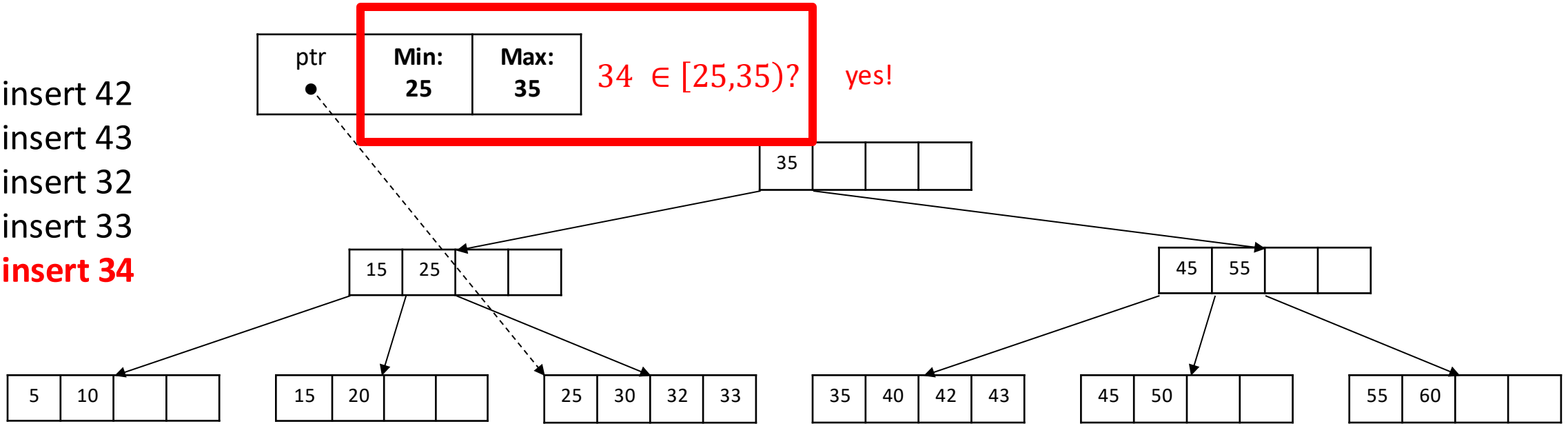


what if we now insert 34?

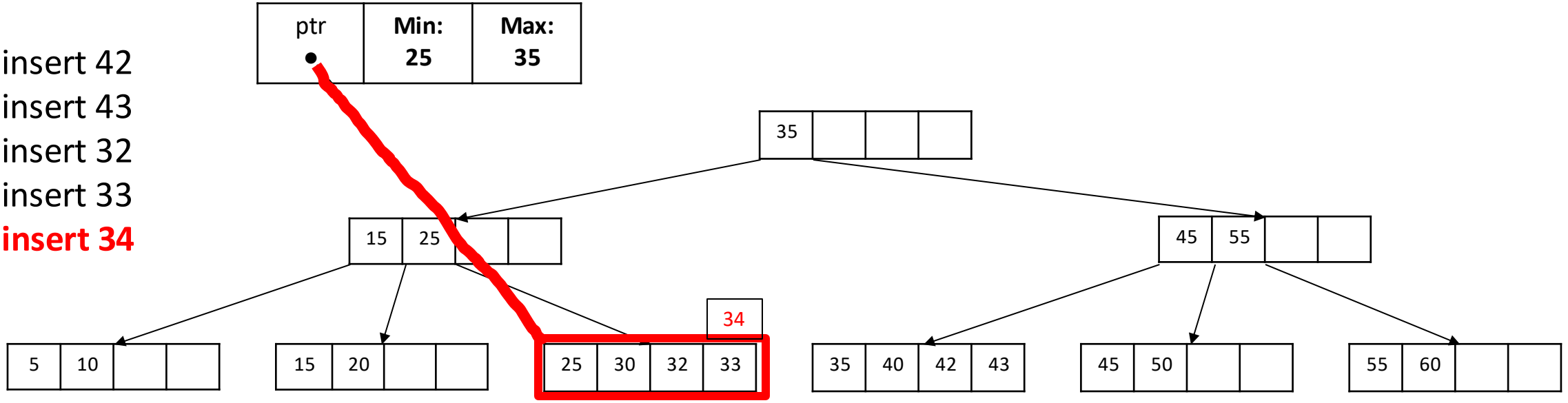
Inserting into the Last Inserted Leaf

insert 42
insert 43
insert 32
insert 33
insert 34



Inserting into the Last Inserted Leaf

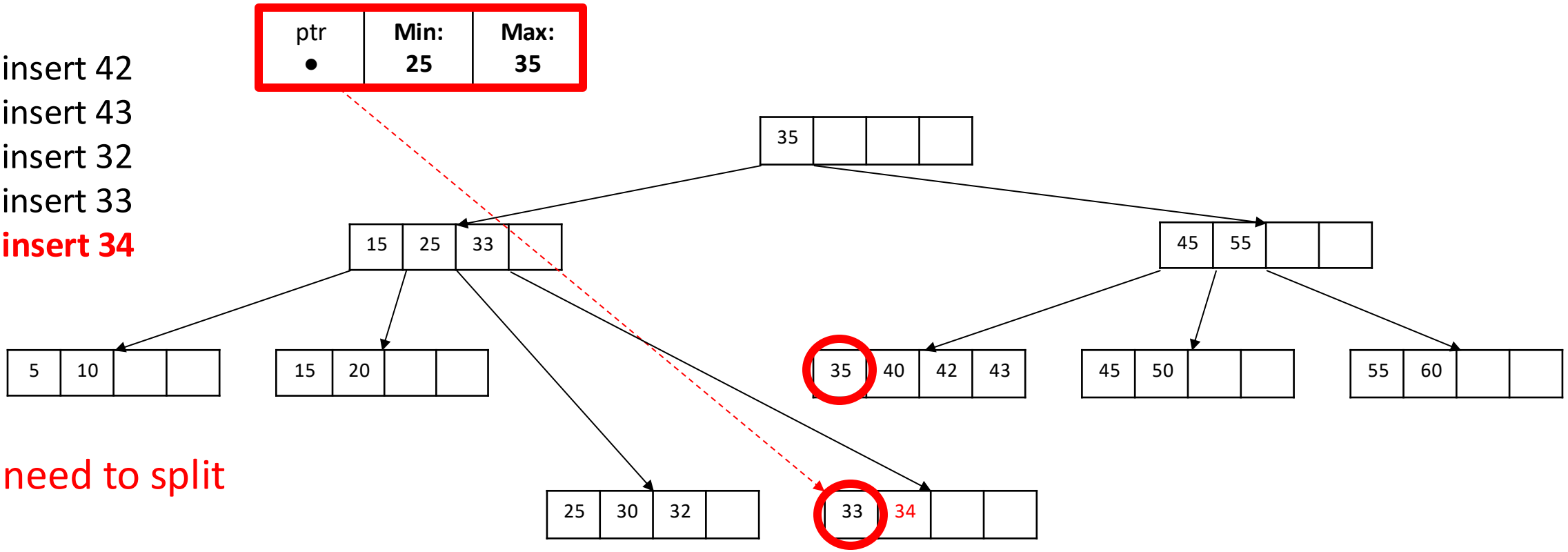
insert 42
insert 43
insert 32
insert 33
insert 34



we need to split

Inserting into the Last Inserted Leaf

insert 42
insert 43
insert 32
insert 33
insert 34



we need to split

lil will follow the node that received the new value

Summary of Loading Options

Option 1: multiple inserts.

- Slow.
- Does not give sequential storage of leaves.

Option 2: *Bulk Loading*

- Fewer I/Os during build.
- Leaves will be stored sequentially (and linked, of course).
- Can control “fill factor” on pages.

Option 3: Optimize Inserts for in-order/near-order insertion

A Note on "Order"

Order (d) concept **replaced** by physical space criterion in practice ("*at least half-full*").

- **Index pages** can typically hold many **more entries** than leaf **pages**.
- **Variable sized records** and search keys mean **different nodes will contain different numbers** of entries.
- Even with fixed length fields, multiple records with the same search key value (*duplicates*) can lead to variable-sized data entries (if we use Alternative (3)).

Many real systems are even sloppier than this --- only reclaim space when a page is *completely* empty.

B+ Trees

"It could be said that the world's information is at our fingertips because of B-trees"

Goetz Graefe

Google (prev. Microsoft, HP Fellow)

ACM Software System Award

Summary

Tree-structured indexes are ideal for range-searches, also good for equality searches.

B+ tree is a dynamic structure.

- Inserts/deletes leave tree height-balanced; $log_F(N)$ cost.
- High fanout (F) means depth rarely more than 3 or 4.
- Almost always better than maintaining a sorted file.
- Typically, 67% occupancy on average.
- If data entries are data records, splits can change rids!