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CS 561: Data Systems Architectures

class 23

Learned Indexes

Prof. Manos Athanassoulis

https://bu-disc.github.io/CS561/

with slides from Jialin Ding

Project Submission & Presentations

April 27th, 11:59pm: *submit preliminary project report & code*

April 28th and May 3rd: *5 + 5 15-minute presentations (12+3 for questions)*
(select your slot in piazza)

May 6th, 11:59pm (hard deadline): *send final report & updated code*

Guest lecture on *"Building a Healthcare Computational Engine: The case for purpose-built systems"*

Angelo Kastroulis, Ballista Technology Group

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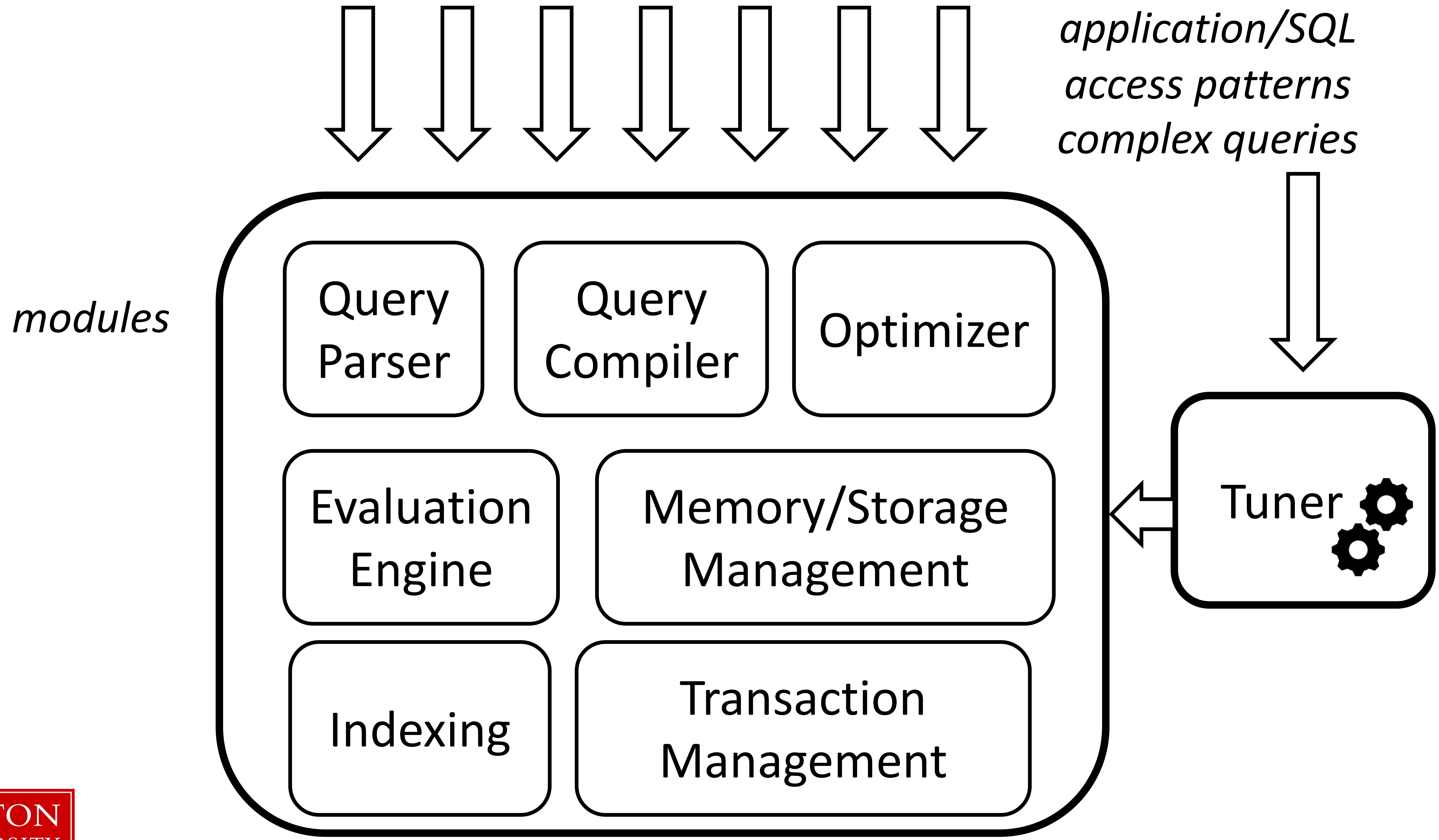
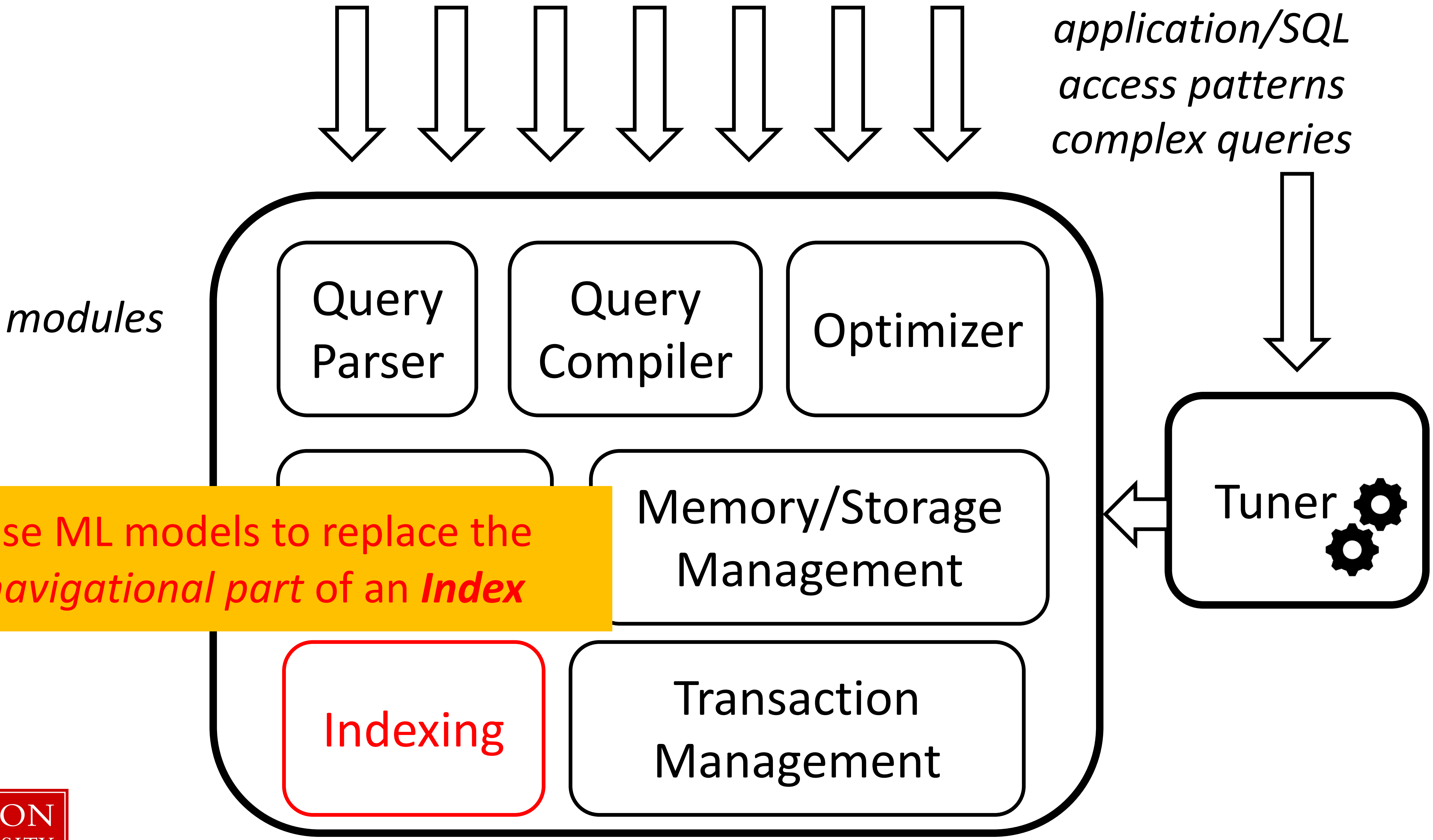
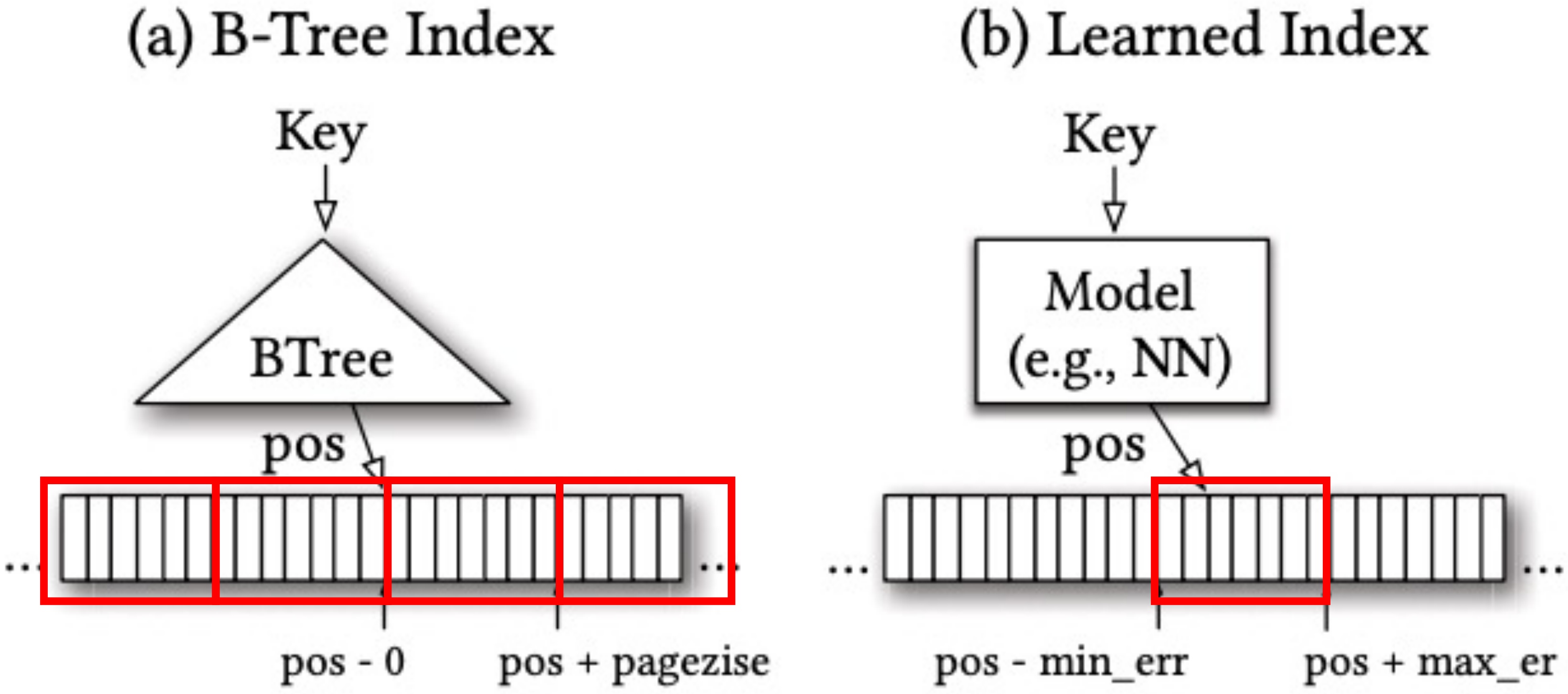

application/SQL
access patterns
complex queries
modules
Query Parser
Query Compiler
Optimizer
Evaluation Engine
Memory/Storage Management
Indexing
Transaction Management
Tuner

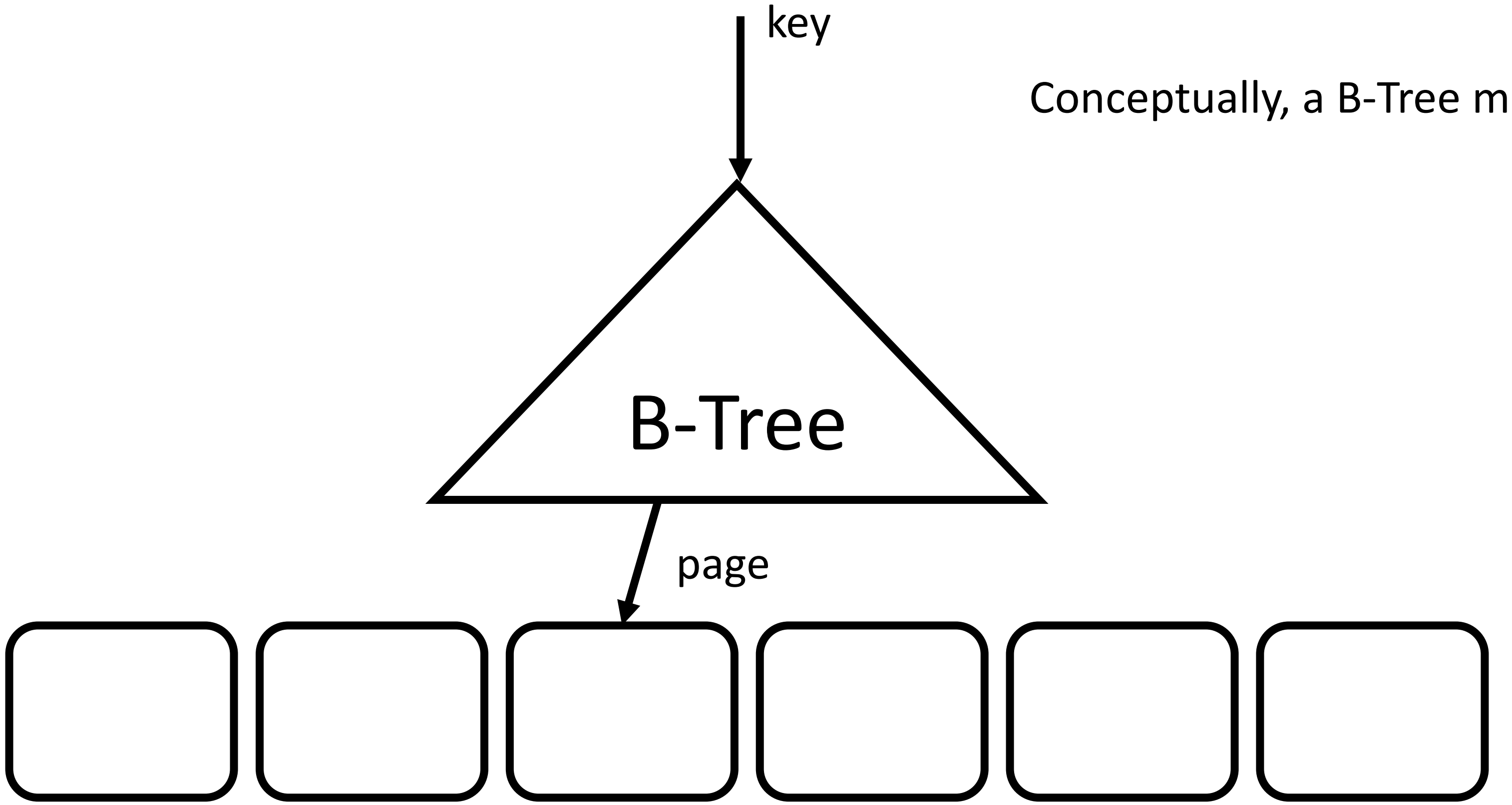
Use ML models to replace the *navigational part* of an ***Index***



B-Trees vs. Learned Indexes

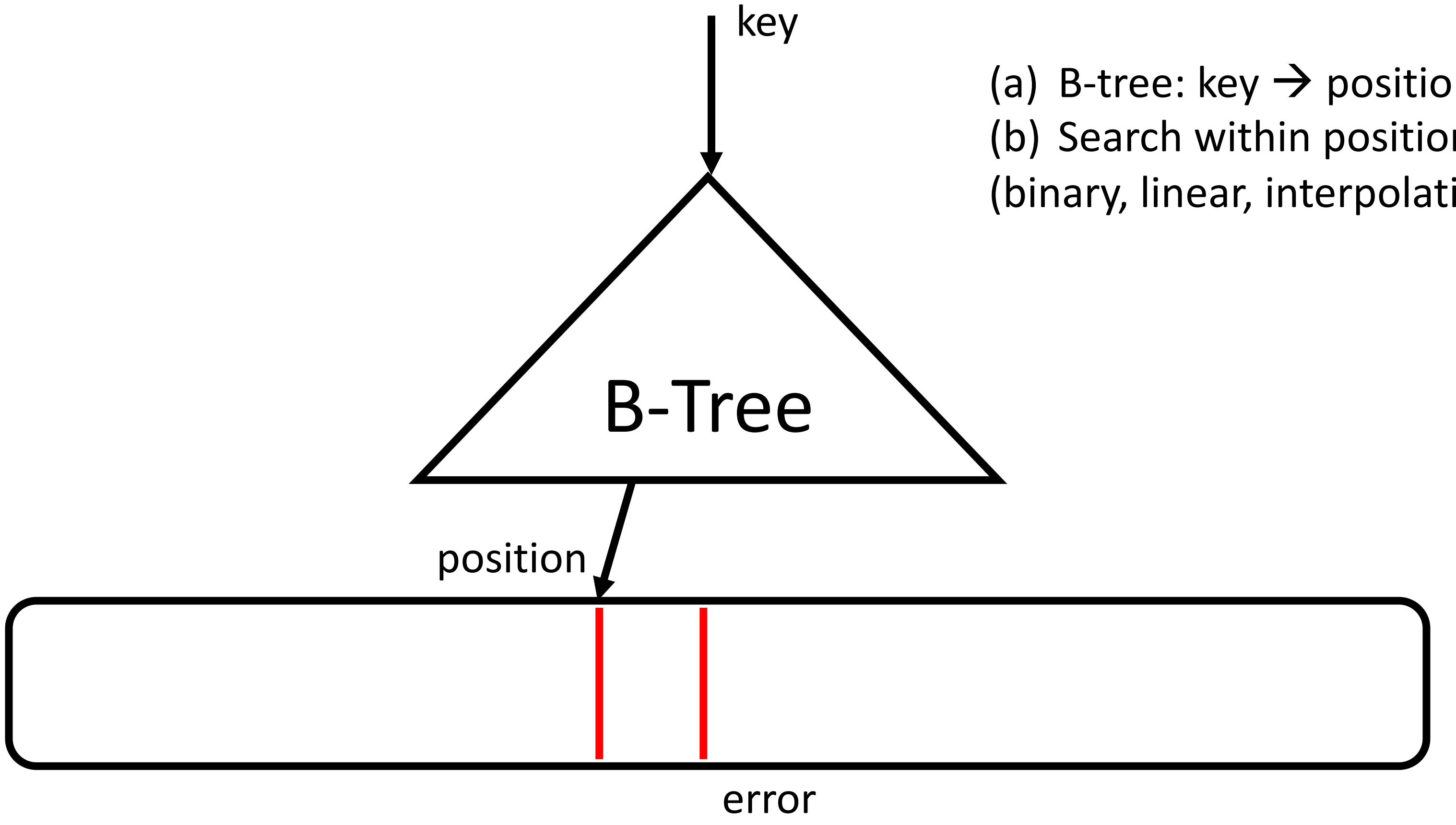


What is the difference?



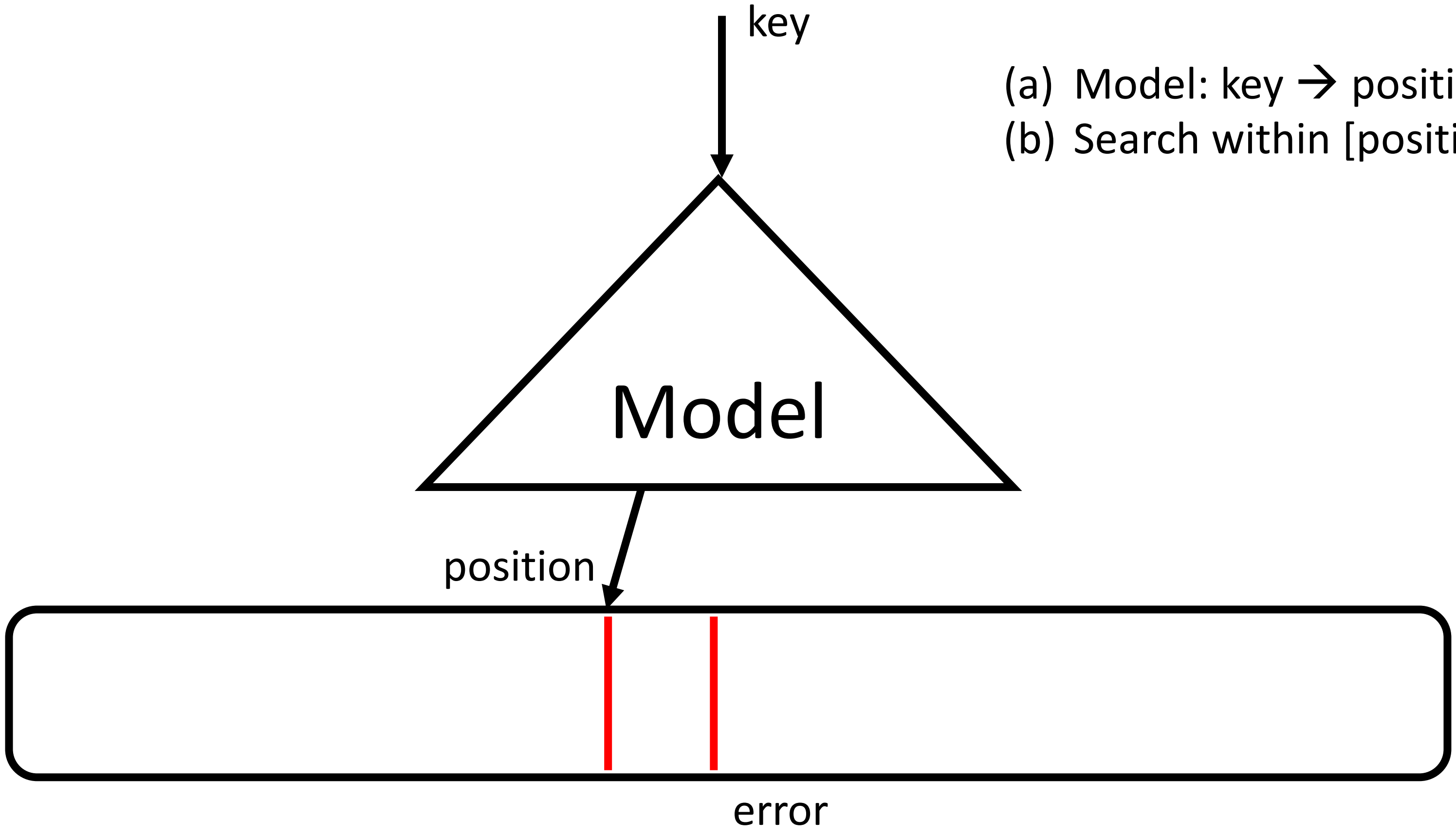
Conceptually, a B-Tree maps a key to a location (page)

Alternative view: data is sorted



(a) B-tree: key → position
(b) Search within position, position+error
(binary, linear, interpolation, exponential search)

A B-Tree is a Model



(a) Model: key → position estimate
(b) Search within [position-error, position+error]

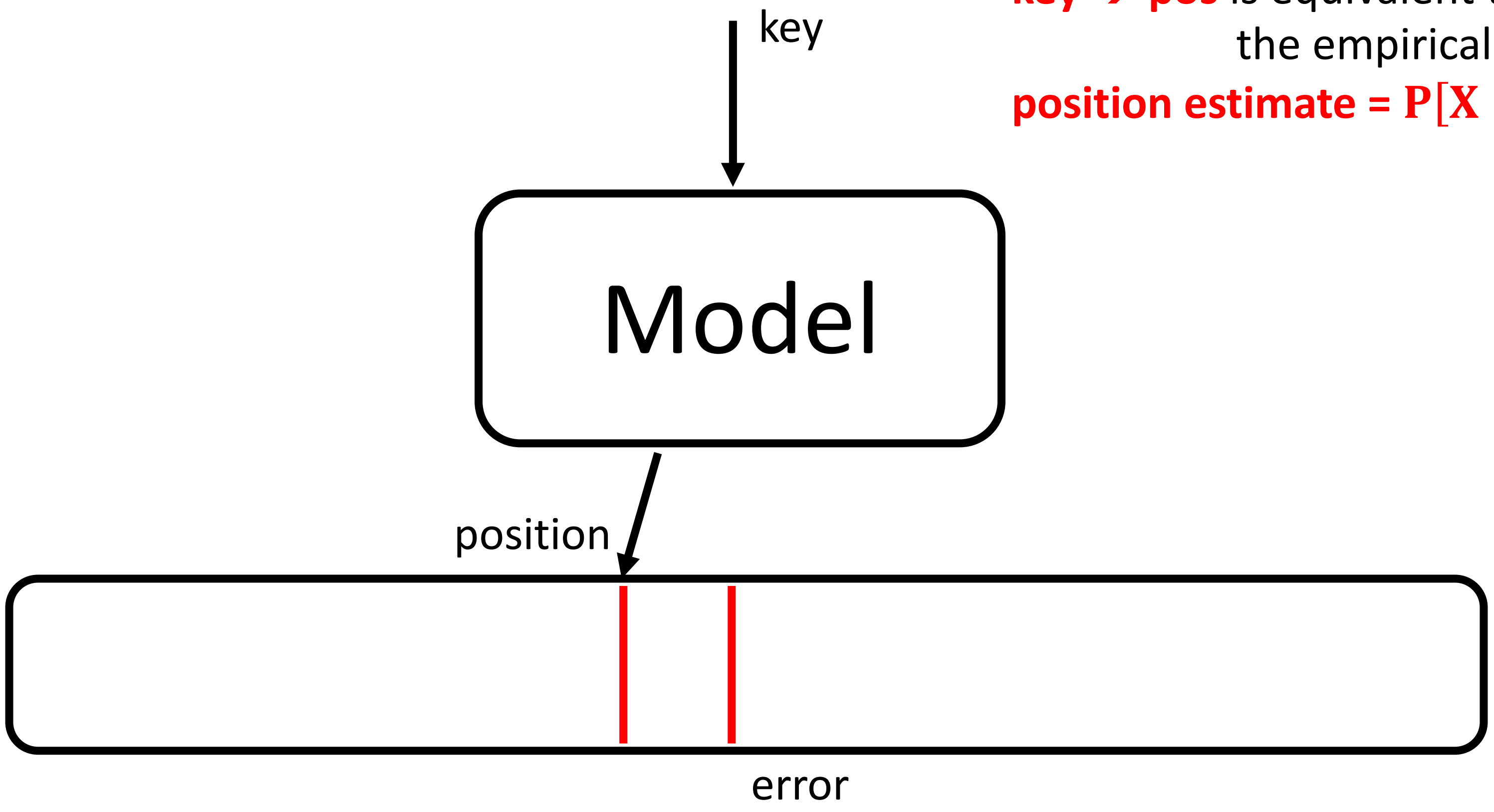
A B-Tree is already a model!

A B-Tree is a Model

A form of regression model

key → pos is equivalent to modeling the empirical CDF of the data

position estimate = $\mathbf{P[X \le \boldsymbol{key}] * \#\boldsymbol{keys}}$



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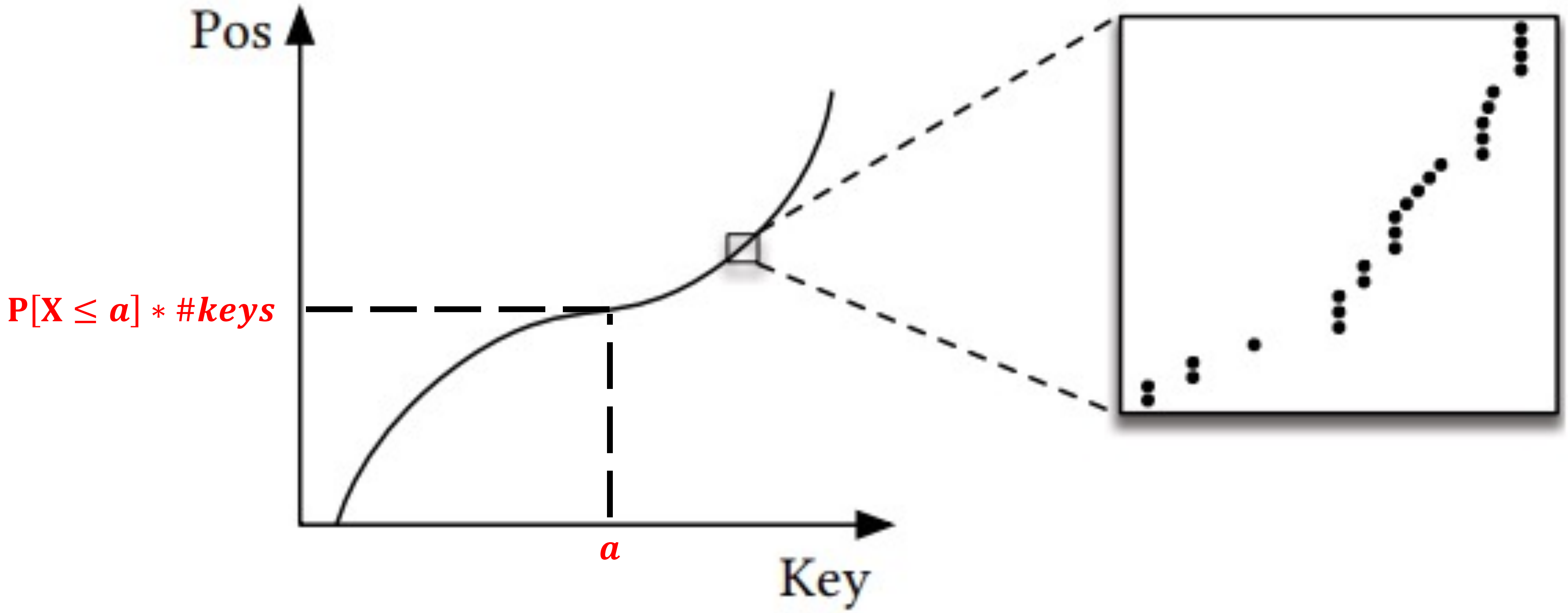
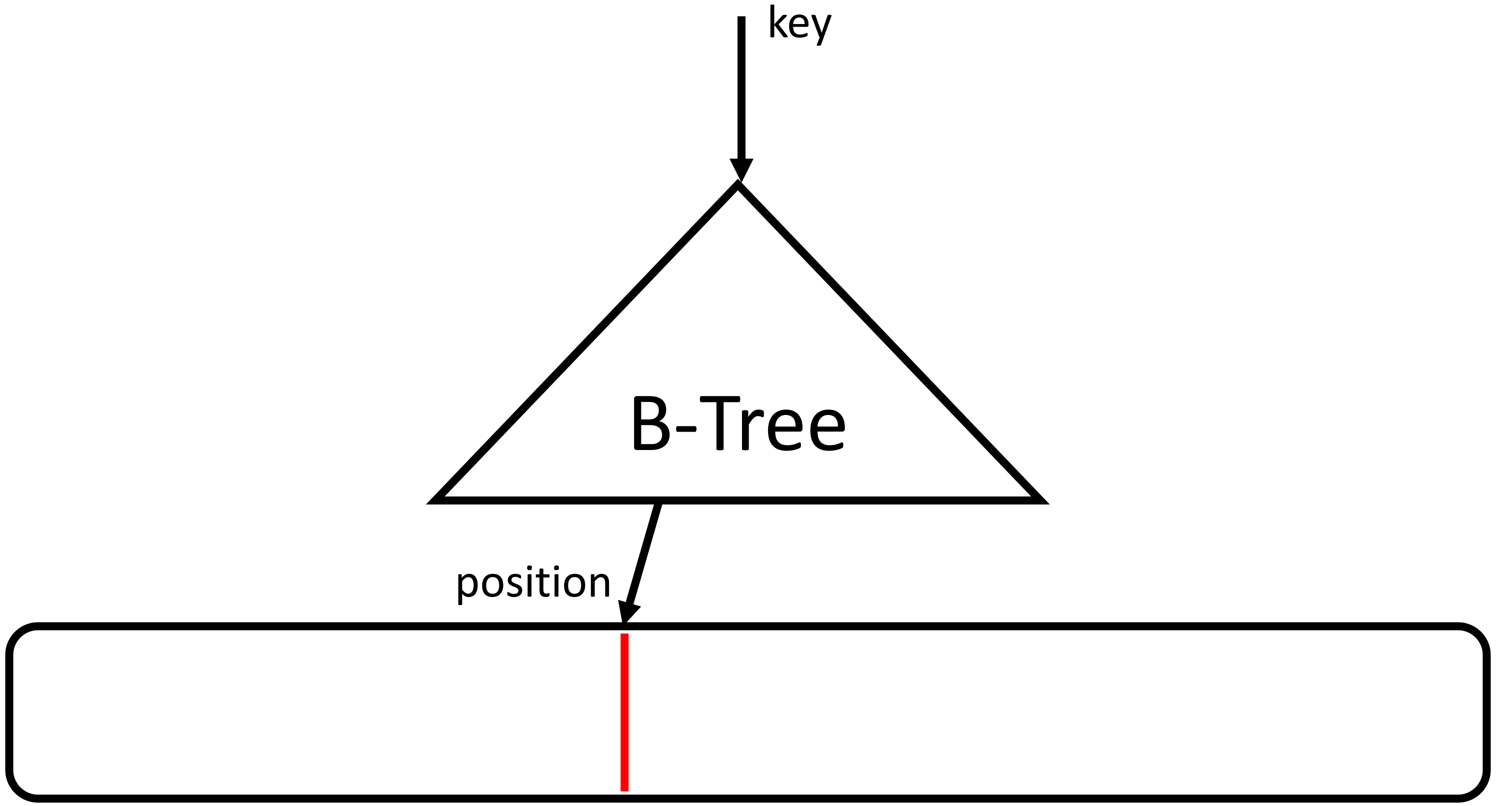

Pos
$P[X \leq a] * \#keys$
a
Key

B-Trees are regression trees



B-Trees is ***already*** a form of a learned index

What does this mean?

Learned Indexes

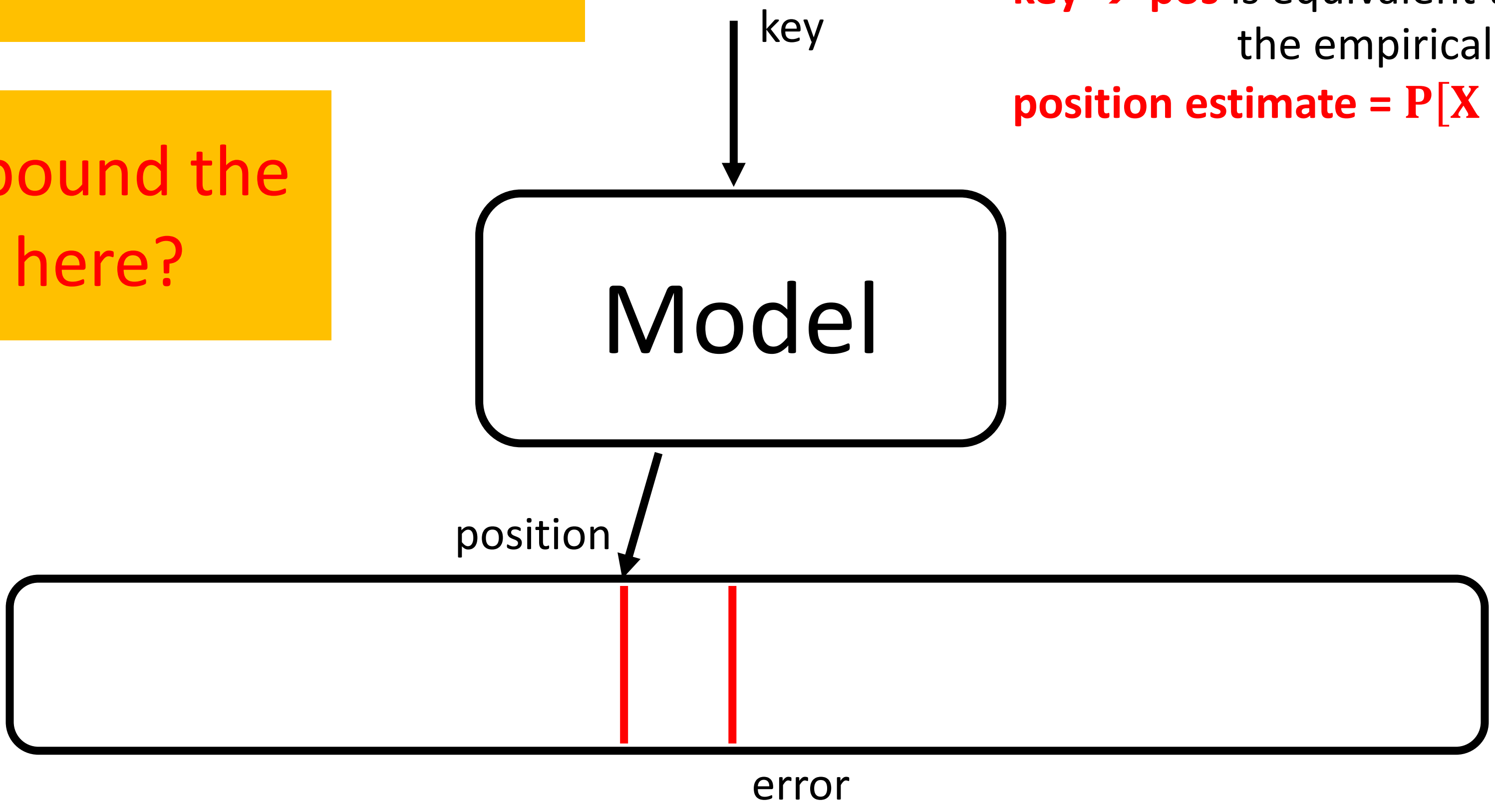
B-Trees have bounded error

Can we bound the error here?

A form of regression model

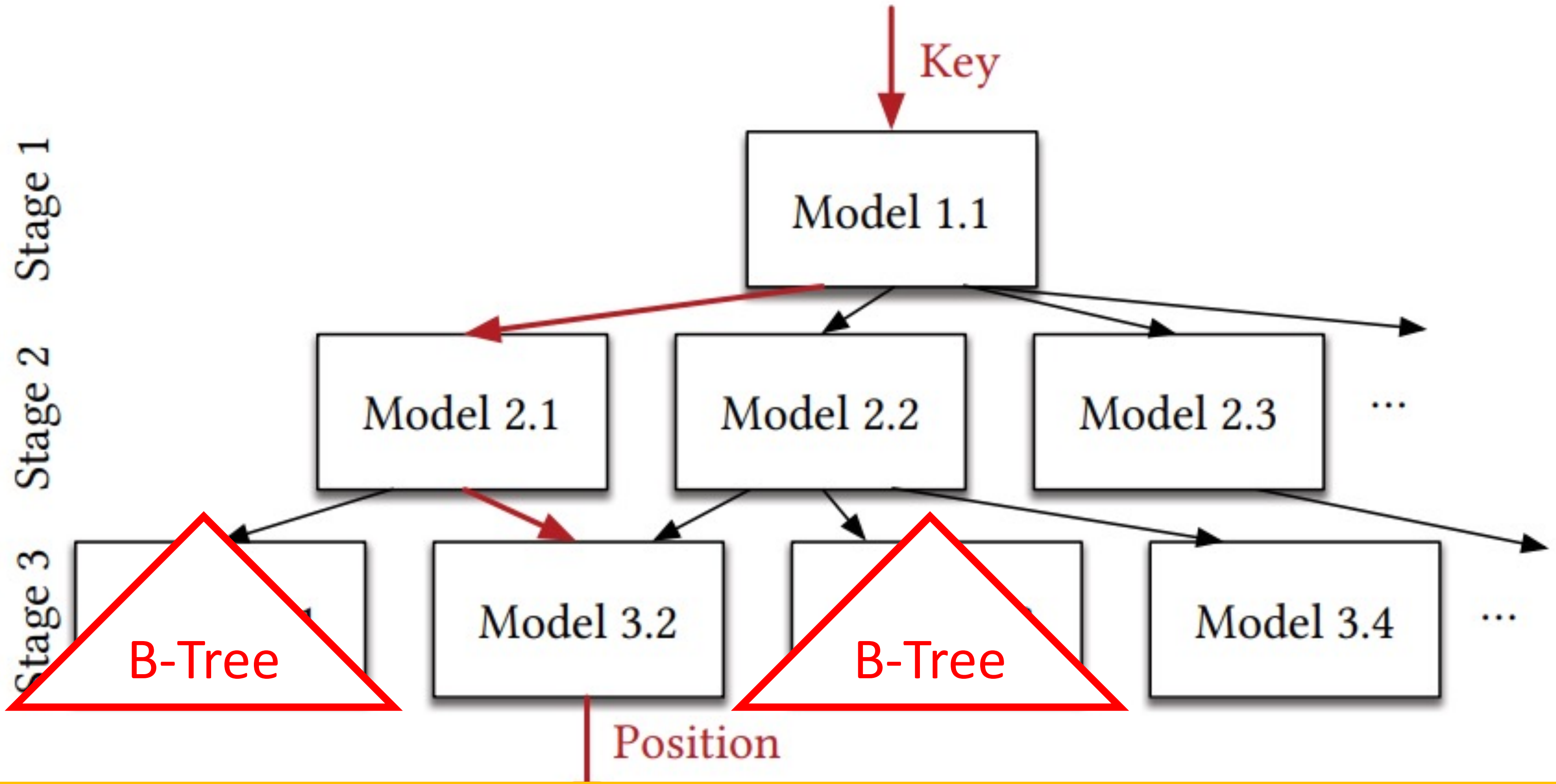
key → pos is equivalent to modeling the empirical CDF of the data

position estimate = $\mathbf{P[X \le \mathit{key}] * \#\mathit{keys}}$



What is the problem if we use an arbitrary model?

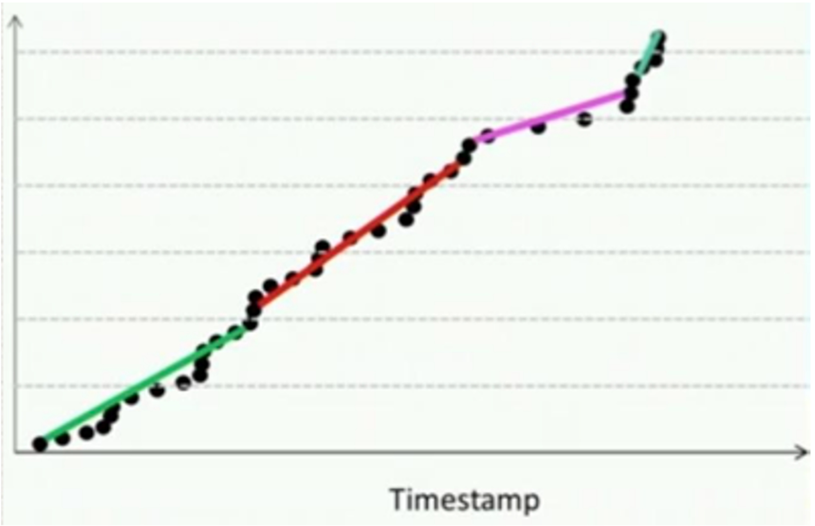
Last-mile indexing



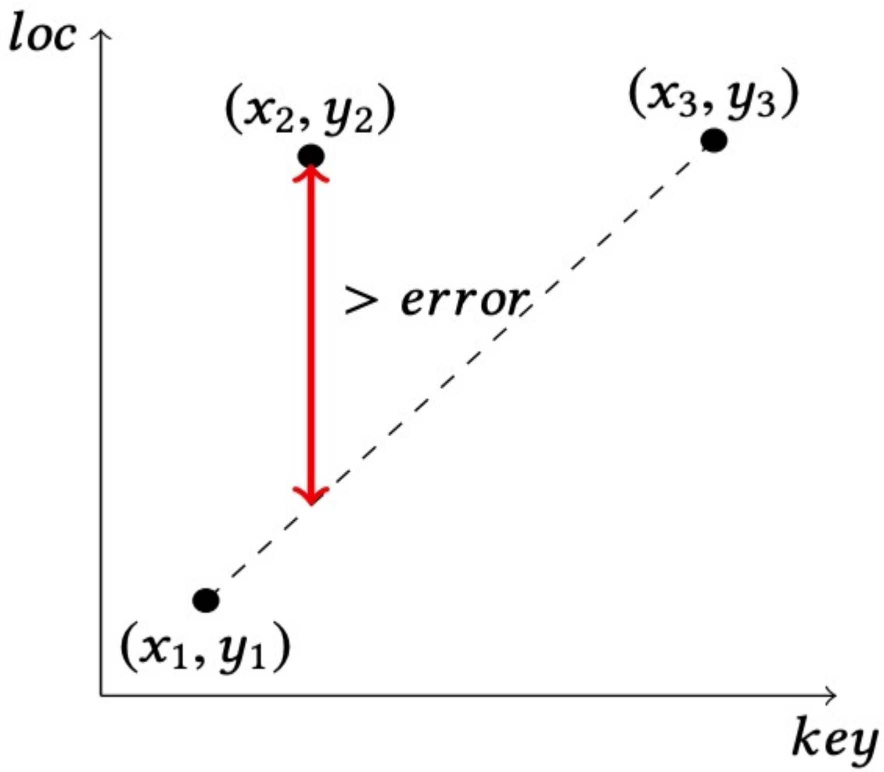
Some models can be replaced sub-B-Trees

Every level provides gain in accuracy

Use case: FITing-Tree

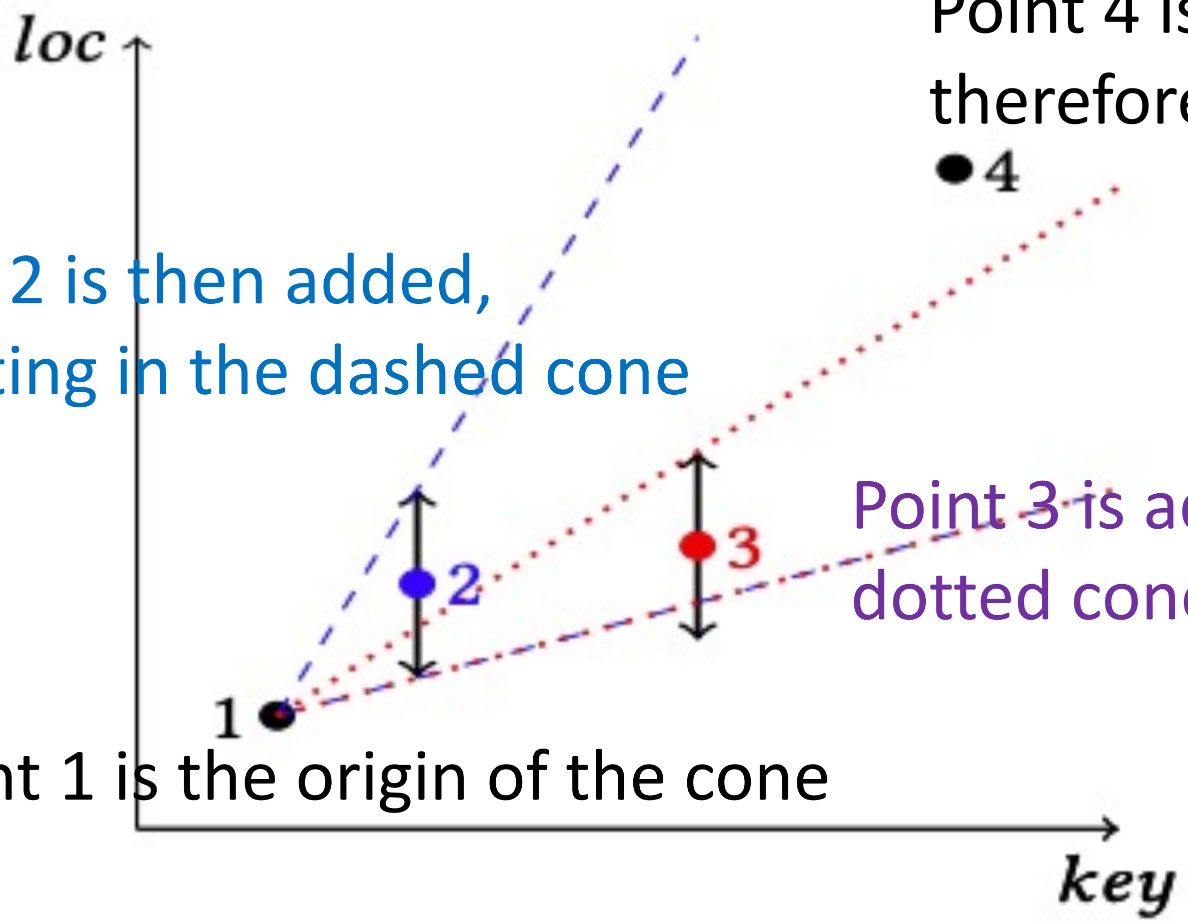


Piece-wise linear approximation



A segment from (x_1,y_1) to (x_3,y_3) is **not valid** if (x_2,y_2) is further than ***error*** from the interpolated line.

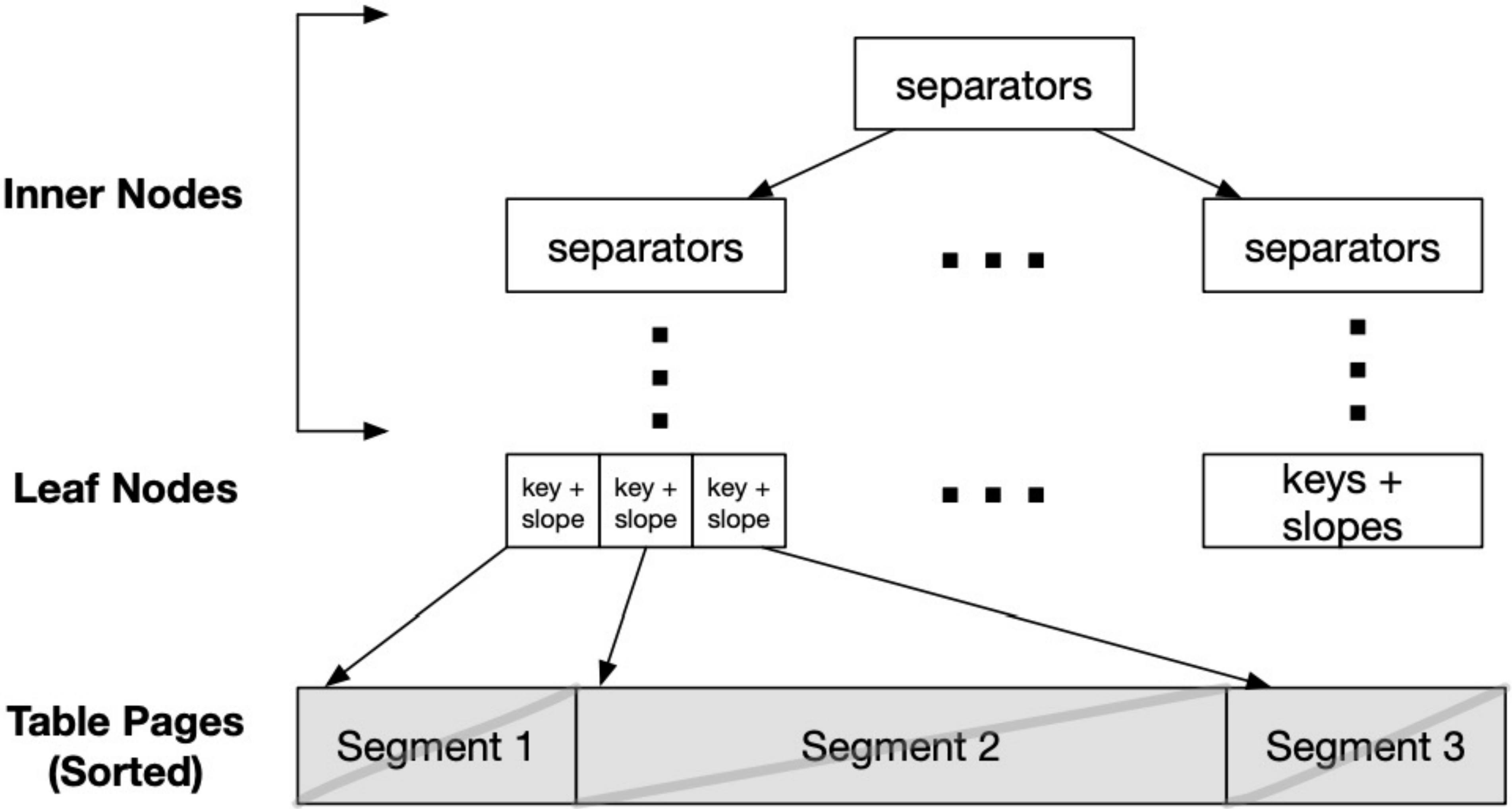
Point 4 is outside the dotted cone and therefore starts a new segment



Point 2 is then added, resulting in the dashed cone

Point 3 is added next, yielding in the dotted cone

Point 1 is the origin of the cone



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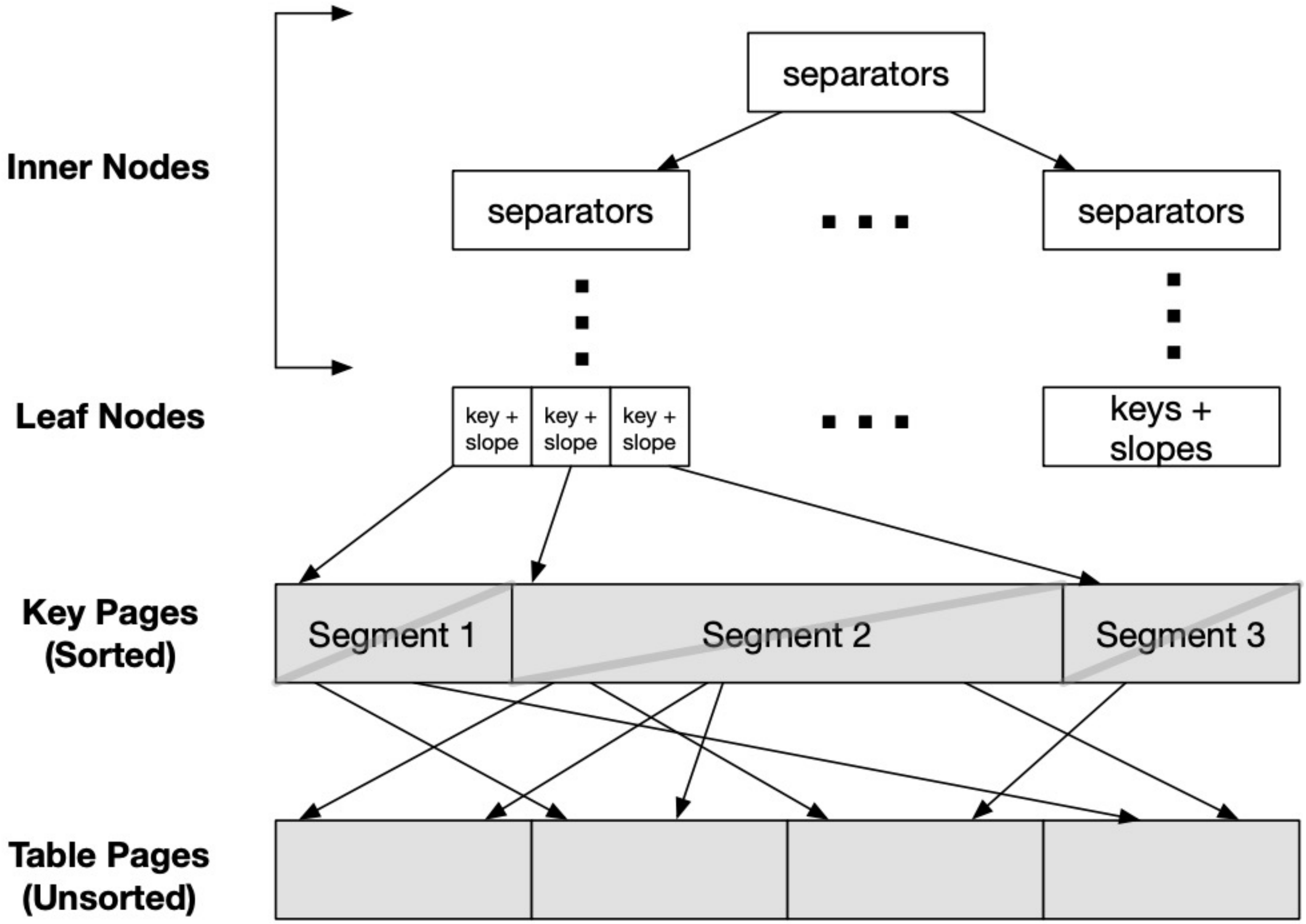
What if base data is not sorted?

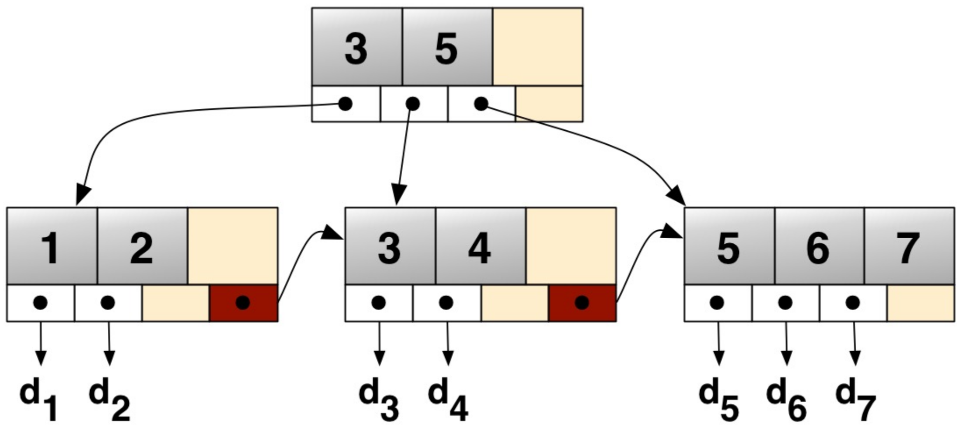


Need to materialize sorted data

What about updates and learned indexes?

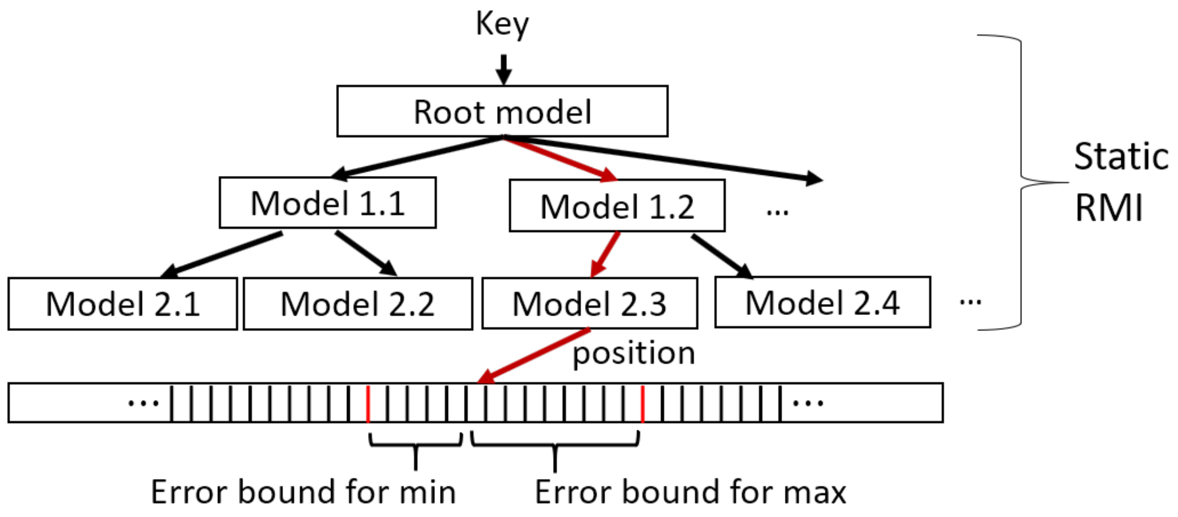
B+ Tree

- Traverses tree using comparisons
- Supports OLTP-style mixed workloads
  - Point lookups, range queries
  - Inserts, updates, deletes



Learned Index (Kraska et al., 2018)

- Traverses tree using computations (models)
- Supports point lookups and range queries
- Advantages: 3X faster reads, 10X smaller size
- Limitation: does not support writes



ALEX goals

	B+ Tree	Learned Index	ALEX
Lookup time	Slow	Fast	Faster
Insert time	Fast	Not Supported	Fast
Space usage	High	Low	Low

(every row should be read independently)

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(every row should be read independently)

ALEX design overview

Structure

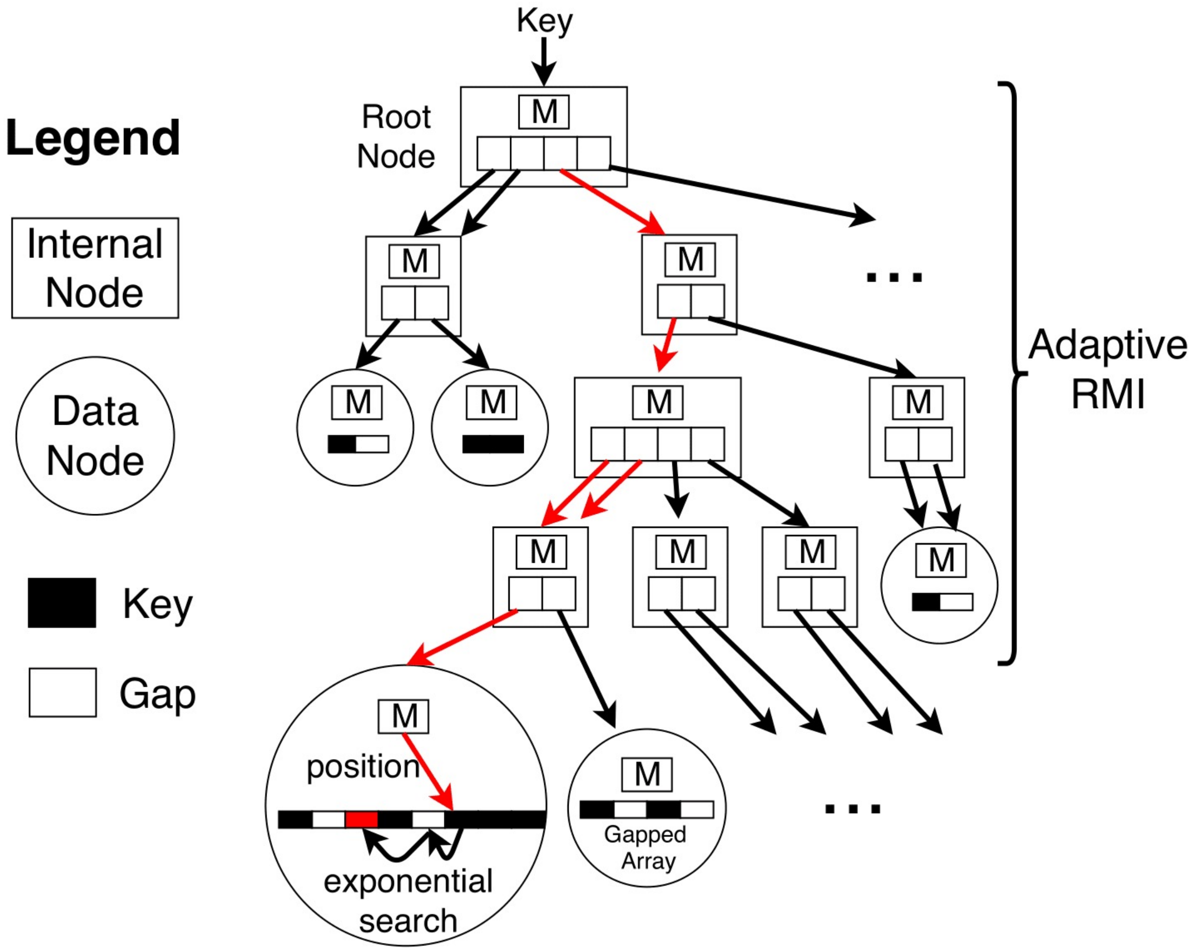
- **Dynamic tree** structure
- Each **node** contains a **<u>linear model</u>**
  - **internal** nodes → **models select the child** node
  - **data** nodes → **models predict the position** of a key

Core operations

- Lookup
  - Use **RMI to predict location of key** in a data node
  - Do **local search to correct** for prediction **error**
- Insert
  - Do a **lookup to find the insert** position
  - **Insert the new key/value** (might require shifting)

Current design constraints

a) In memory
b) Numeric data types
c) Single threaded



ALEX Core Ideas

	Faster Reads	Faster Writes	Adaptiveness
1. Gapped Array		✓	
2. Model-based Inserts	✓		
3. Exponential Search	✓		
4. Adaptive Tree Structure	✓	✓	✓

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1. Gapped Array

How should data be stored in data nodes?

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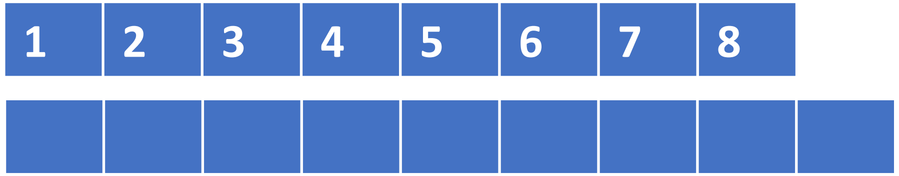
1. Gapped Array

Dense Array

1	2	3	4	5	6	7	8

1. Gapped Array

Dense Array



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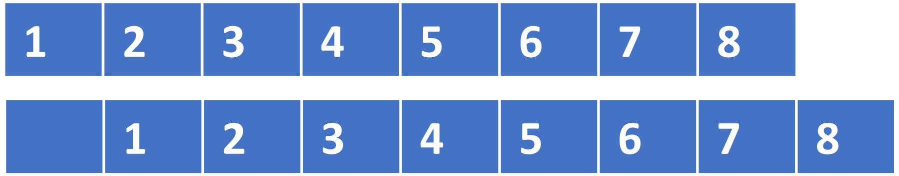
1. Gapped Array

Dense Array



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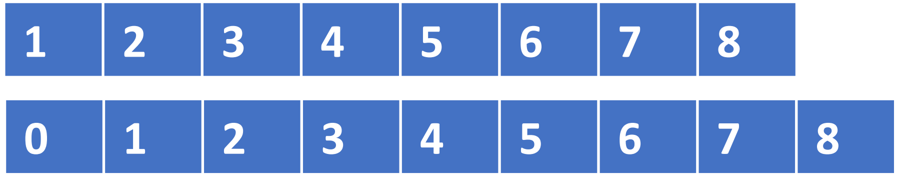
1. Gapped Array

Dense Array



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1. Gapped Array

Dense Array

0	1	2	3	4	5	6	7	8

1. Gapped Array

Insertion Time

Dense Array

0	1	2	3	4	5	6	7	8

$O(n)$

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1. Gapped Array

Insertion Time

Dense Array

$O(n)$

B+ Tree Node

1. Gapped Array

Insertion Time

Dense Array

0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

$O(n)$

B+ Tree Node

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**			

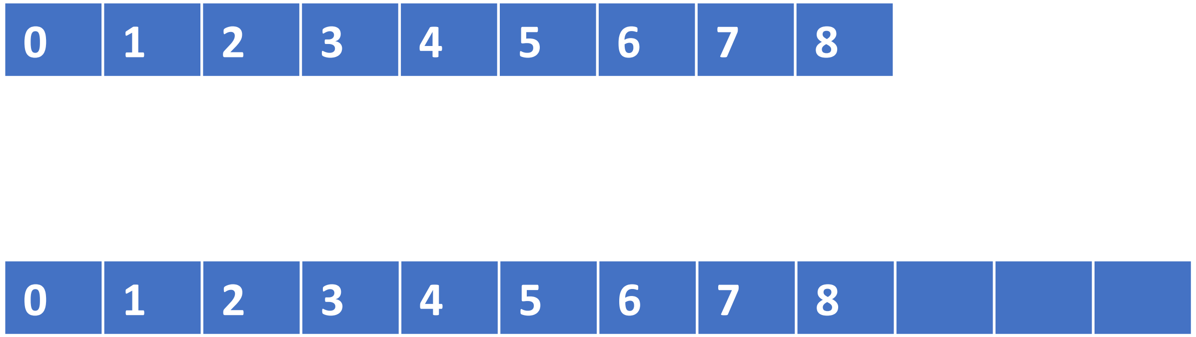
1. Gapped Array

Insertion Time

Dense Array

$O(n)$

B+ Tree Node



1. Gapped Array

Insertion Time

Dense Array

0	1	2	3	4	5	6	7	8

$O(n)$

B+ Tree Node

0	1	2	3	4	5	6	7	8			

$O(n)$

1. Gapped Array

													Insertion Time
Dense Array	0	1	2	3	4	5	6	7	8				$O(n)$
B+ Tree Node	0	1	2	3	4	5	6	7	8				$O(n)$
Gapped Array	1	2		3	4		5	6		7	8		

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Dense Array	0	1	2	3	4	5	6	7	8				$O(n)$
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Gapped Array	0	1	2	3	4		5	6		7	8		$O(\log n)$

1. Gapped Array

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Storing data in Gapped Arrays achieves inserts using fewer shifts, leading to faster writes

1. Gapped Array

													Insertion Time
Dense Array	0	1	2	3	4	5	6	7	8				$O(n)$
B+ Tree Node	0	1	2	3	4	5	6	7	8				$O(n)$
Gapped Array	0	1	2	3	4	5	5	6	7	7	8	S	$O(\log n)$

Storing data in Gapped Arrays achieves inserts using fewer shifts, leading to faster writes

2. Model-based Inserts

Where do we put gaps in the Gapped Array?

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2. Model-based Inserts

Gapped Array

1	2		3	4		5	6		7	8	

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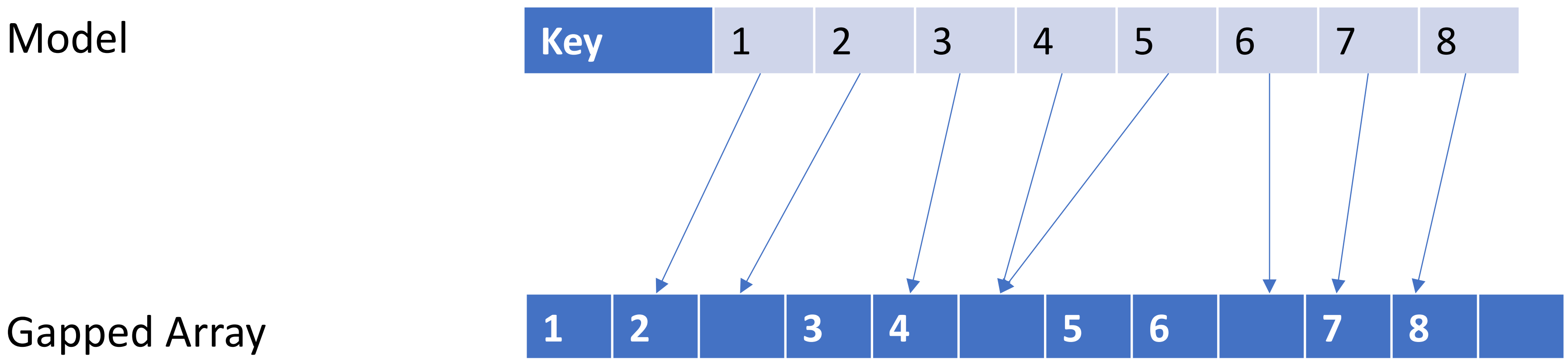
2. Model-based Inserts



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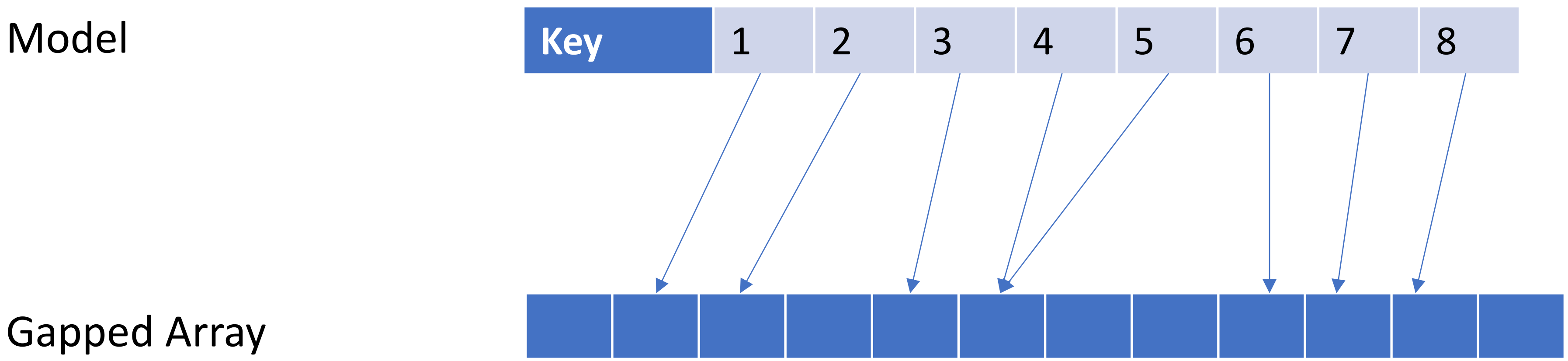

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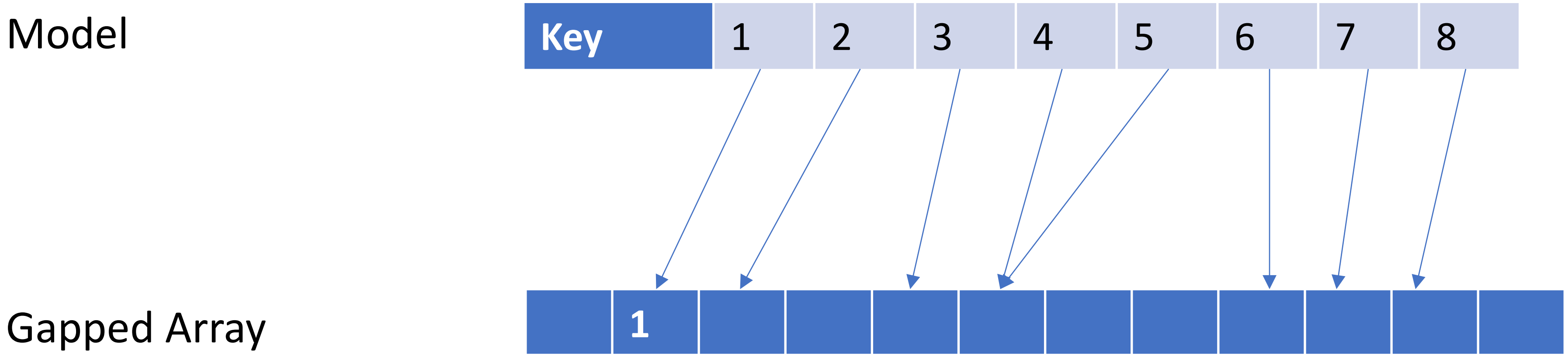
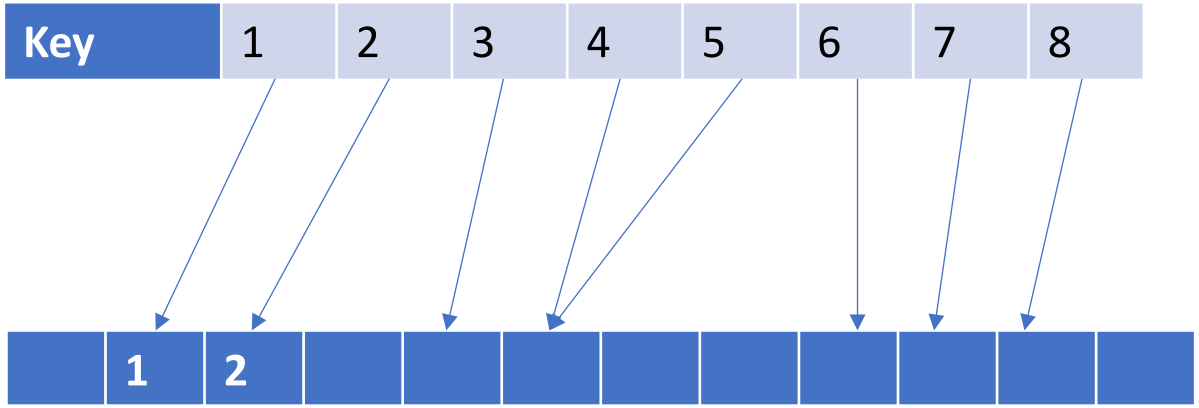


2. Model-based Inserts

Model

Gapped Array



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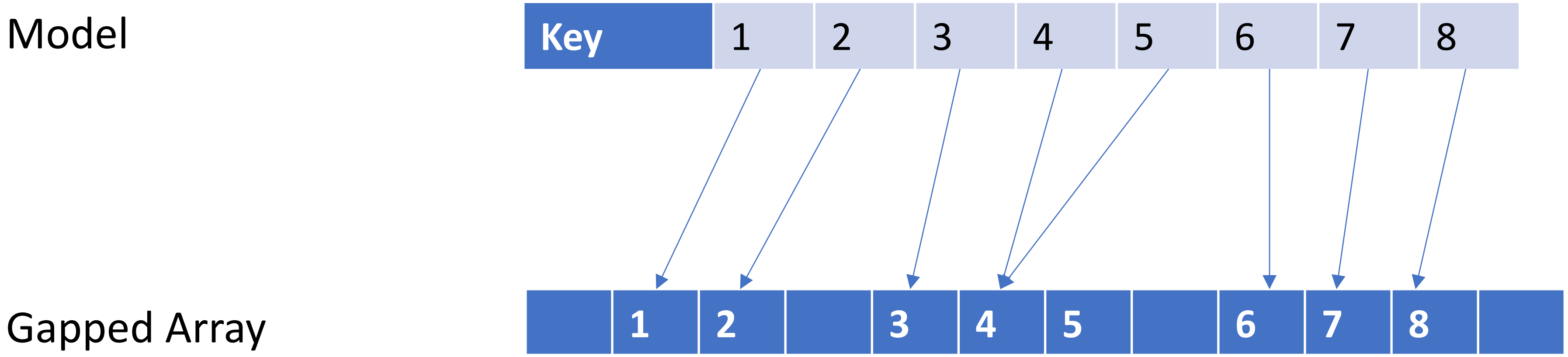

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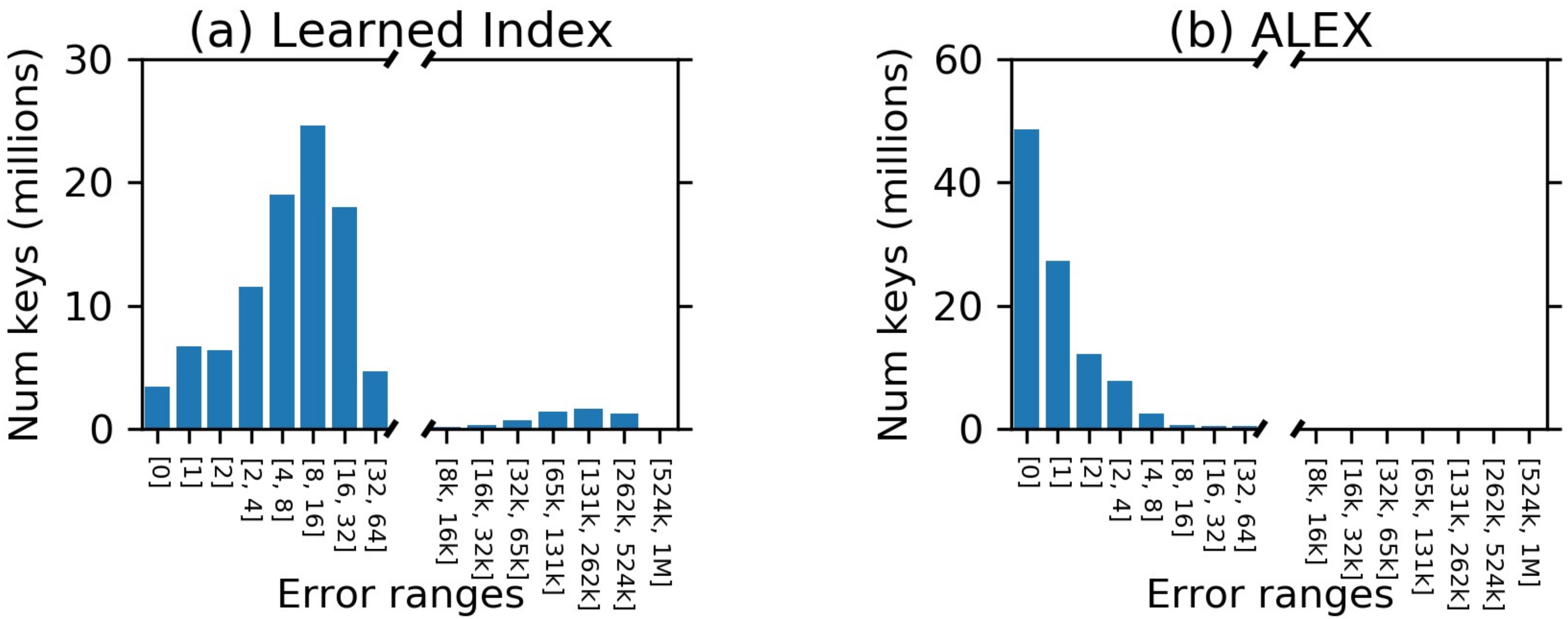

Model-based inserts achieve lower prediction error, leading to faster reads

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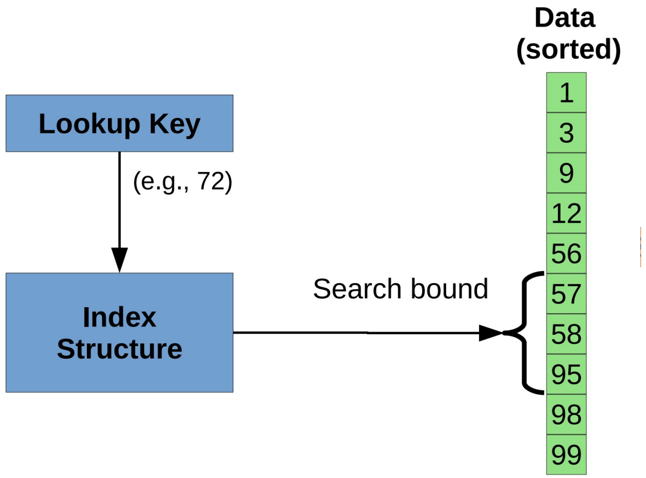
3. Exponential Search



Can we do better than binary search?

Explanation: Exponential Search

1	2	3	5	6	7	8	13	15	17	18	21	22	23

```
int exponential_search(T arr[], int size, T key)
{
    if (size == 0) {
        return NOT_FOUND;
    }

    int bound = 1;
    while (bound < size && arr[bound] < key) {
        bound *= 2;
    }

    return binary_search(arr, key, bound/2, min(bound + 1, size));
}
```


Core algorithm: binary search!

Key difference: exp. increasing search bound

Explanation: Exponential Search

1	2	3	5	6	7	8	13	15	17	18	21	22	23

search for 3

```
int exponential_search(T arr[], int size, T key)
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Explanation: Exponential Search

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Explanation: Exponential Search

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search for 22

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Explanation: Exponential Search

1	2	3	5	6	7	8	13	15	17	18	21	22	23

search for 22

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Explanation: Exponential Search

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Explanation: Exponential Search

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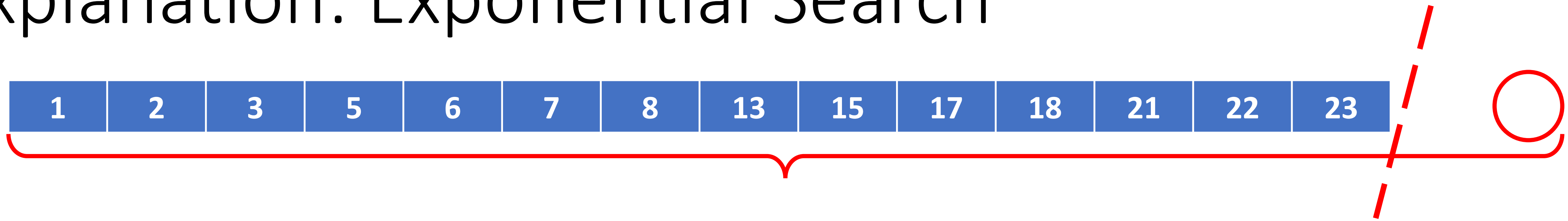
Explanation: Exponential Search



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Explanation: Exponential Search

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Explanation: Exponential Search

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```
int exponential_search(T arr[], int size, T key)
{
    if (size == 0) {
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    }
```

We begin our search from the "predicted" location, ***low error expected***!

```
        bound *= 2;
    }
```

Why is this helpful in our case?

```
}
```

Exp. Search is ***ideal*** for a **search key at the beginning** of the array!

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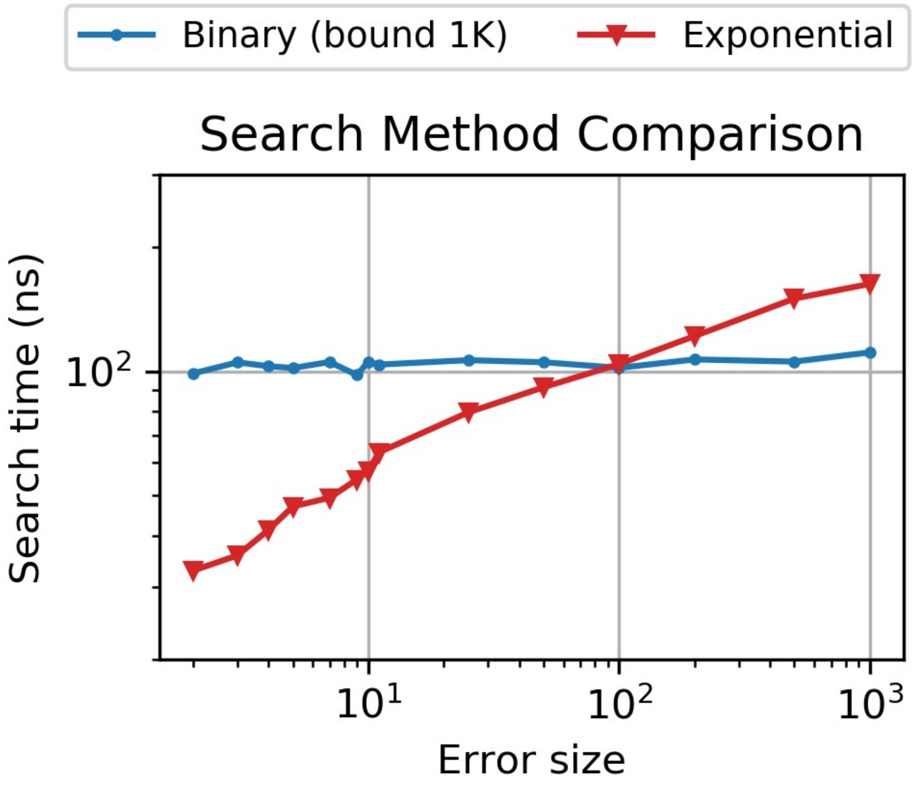
3. Exponential Search



Model errors are low, so exponential search is faster than binary search

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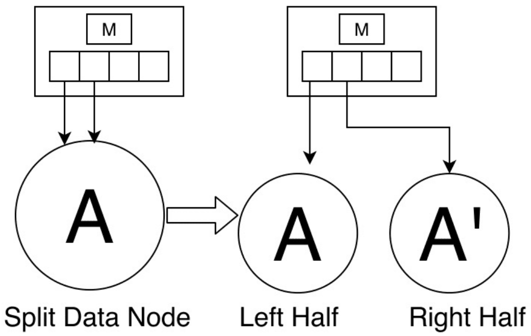
4. Adaptive Structure

What happens if data nodes become full?

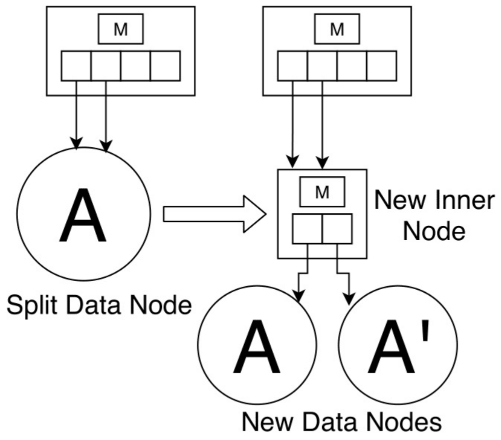
What happens if models become inaccurate?

4. Adaptive Structure

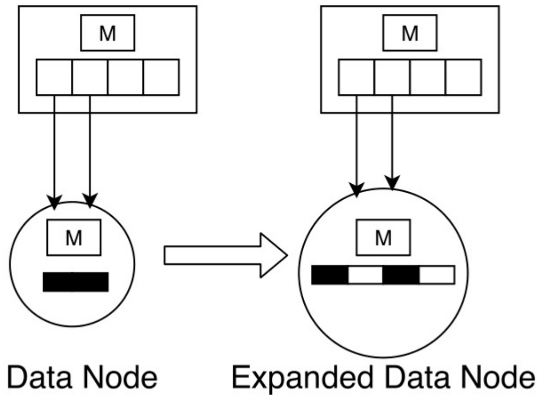
- Flexible tree structure
  - Split nodes sideways
  - Split nodes downwards
  - Expand nodes
  - Merge nodes, contract nodes
- Key idea: all decisions are made to maximize performance
  - Use cost model of query runtime
  - No hand-tuning
  - Robust to data and workload shifts



(a)

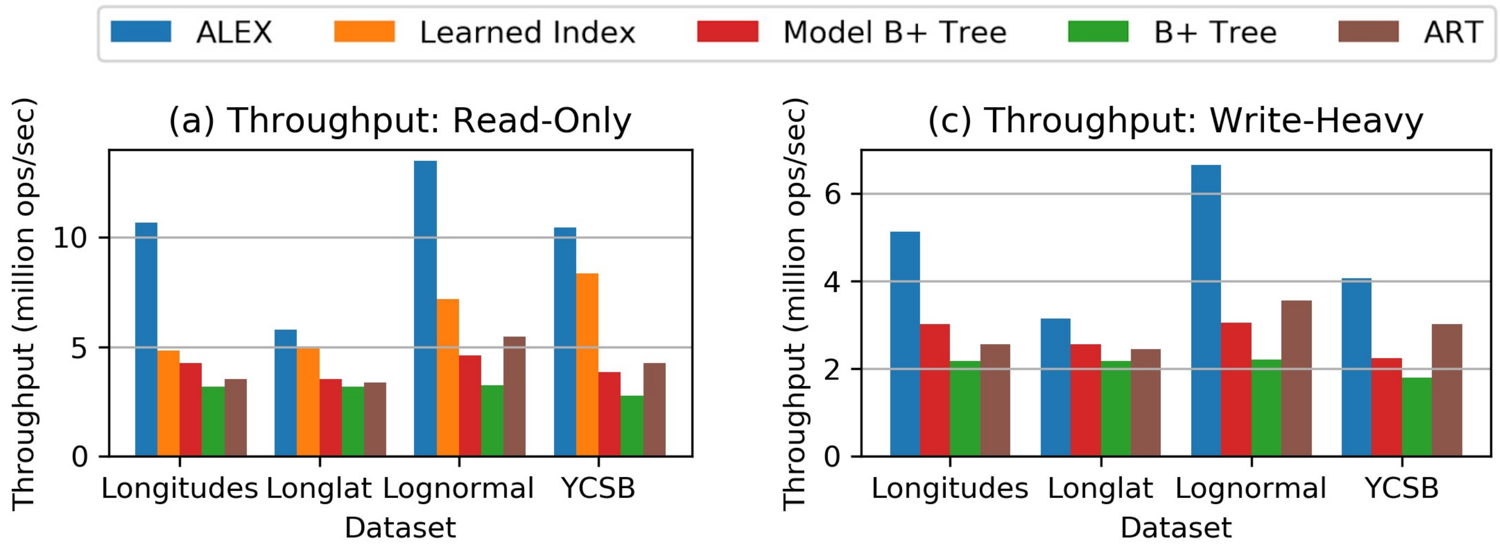


(c)



Results

- High-level results
  - Fast reads
  - Fast writes

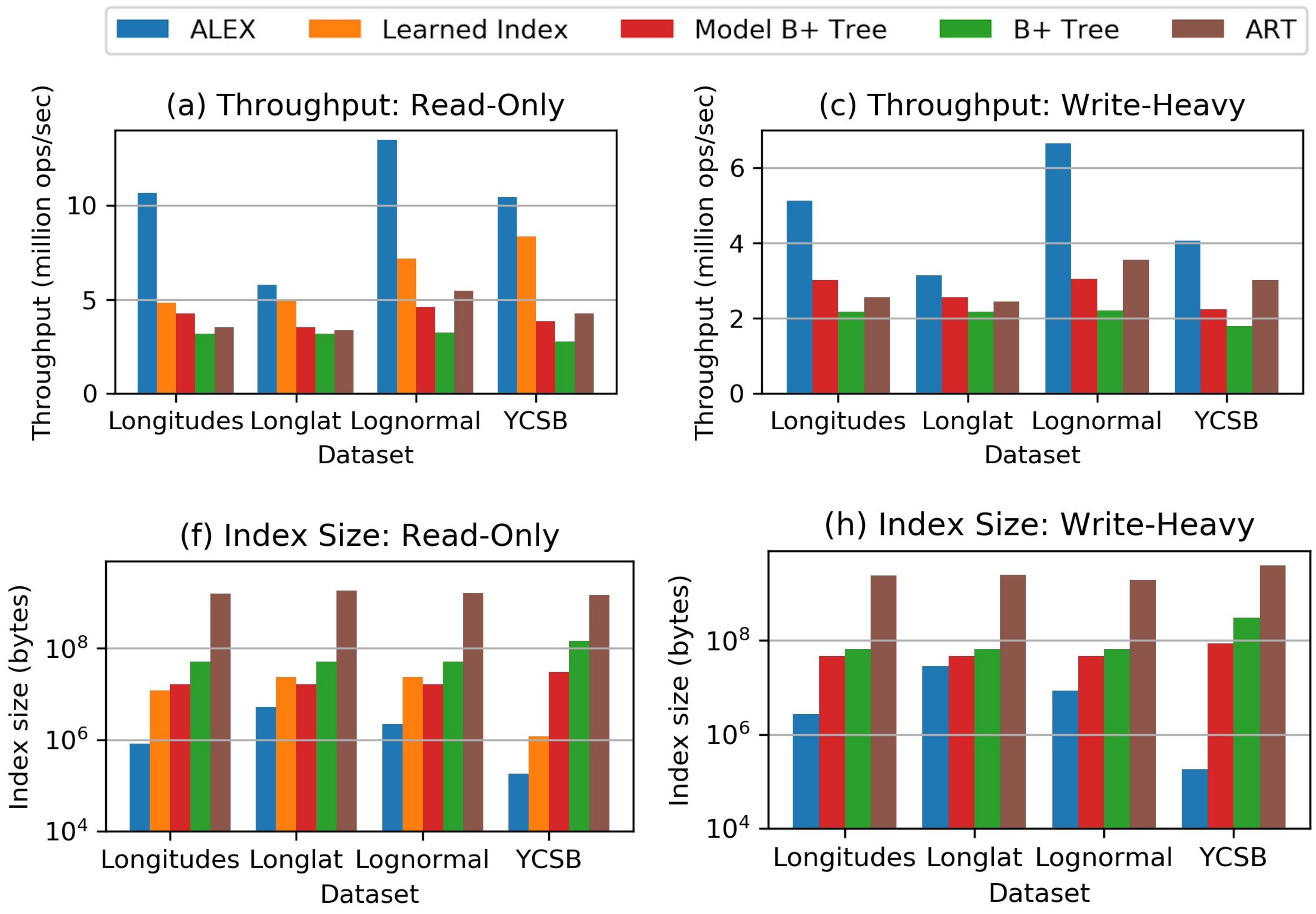


~4x faster than B+ Tree
~2x faster than Learned Index

~2-3x faster than B+ Tree

Results

- High-level results
  - Fast reads
  - Fast writes
  - Smaller index size
- Other results
  - Efficient bulk loading
  - Scales
  - Robust to data and workload shift



~3 orders of magnitude less space for index

ALEX Summary

- Combines the best of B+ Tree and Learned Indexes
  - Supports OLTP-style mixed workloads
    - Point lookups, range queries
    - Inserts, updates, deletes
  - Up to 4X faster, 2000X smaller than B+ Tree
- Current research
  - String keys
  - Concurrency
  - Persistence

	Faster Reads	Faster Writes	Adaptiveness
Gapped Array		✓	
Model-based Inserts	✓		
Exponential Search	✓		
Adaptive Tree Structure	✓	✓	✓

github.com/microsoft/ALEX

Learned Indexes

Replace data structure with **learned models**

- ✓ Simple approaches like linear approximation work well
- ✓ Empty space for updates
- ✓ Error bounds to split model nodes
- ✓ Exponential search for last-mile searching

- ➢ A very fertile area of research!
- ➢ A comprehensive list of papers: http://dsg.csail.mit.edu/mlforsystems/papers/#learned-range-indexes

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CS 561: Data Systems Architectures

class 23

Learned Indexes

Prof. Manos Athanassoulis

https://bu-disc.github.io/CS561/